Higher Physics: Mechanics and the properties of matter

Summary Notes

Vectors

A physical quantity is a measurable quantity. Physics is about the relationships between physical quantities.
There are two kinds of physical quantity. One consists of a number of units, the other consists of a number of units in a direction. They have to be recognised and treated differently.

Scalar A scalar quantity is a physical quantity which is completely defined by its magnitude.
Scalar quantities are combined using simple arithmetic.

Vector A vector quantity is a physical quantity which is completely defined by its magnitude and direction.
Vector quantities are added together in triangles.

N

5 km

10 N
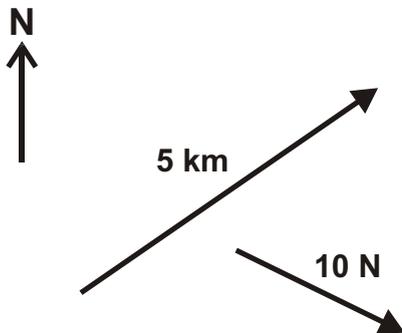
Vectors are represented in magnitude (scaled) and direction by arrows.

SCALARS	VECTORS
distance	*displacement*
speed	*velocity*
mass	*acceleration*
energy	*force*
time	*momentum*

Distance / displacement   **distance :** A distance is simply the length of a journey.

displacement : displacement is the distance moved in a given direction.

Speed / velocity   **speed :** The speed of an object is the rate at which distance is increasing.

velocity : velocity is the rate at which displacement is changing. Velocity should always be quoted with a direction.

$$\text{Average speed} \ = \ \frac{\textbf{total distance covered}}{\textbf{time taken}}$$

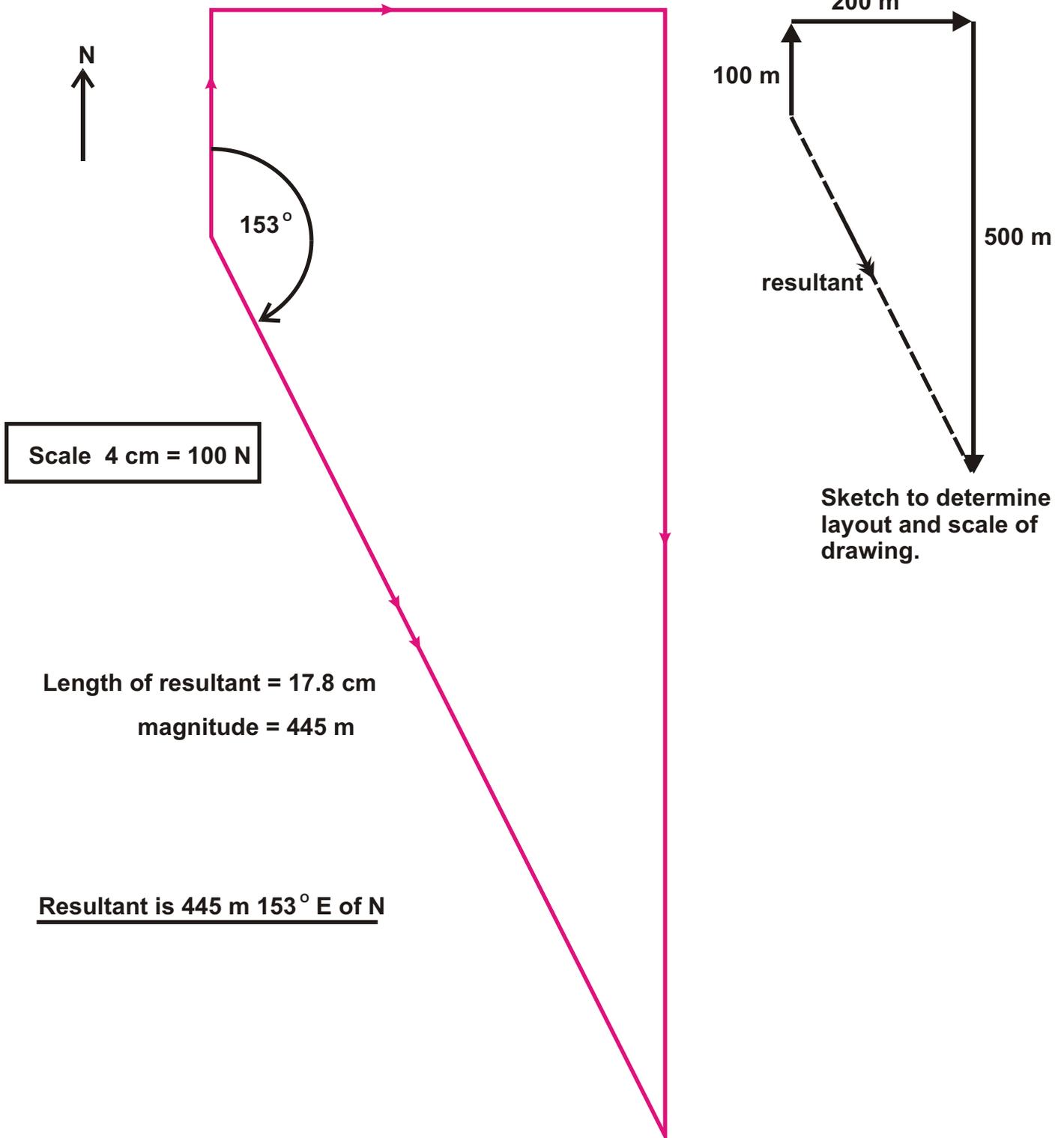$$\text{Average velocity} = \ \frac{\textbf{resultant displacement}}{\textbf{time taken}}$$

Finding the Resultant vector.

The combined effect of a number of vectors can be found by adding the vectors together.
The result of the addition is the **resultant vector.**
The resultant vector is a single vector which represents the combined effect of a number
of vectors and can be used to calculate the effect of those vectors.

Example 1. *Find the resultant displacement of a journey consisting of the following*
displacements: 100 m North; 200 m East; 500 m South.
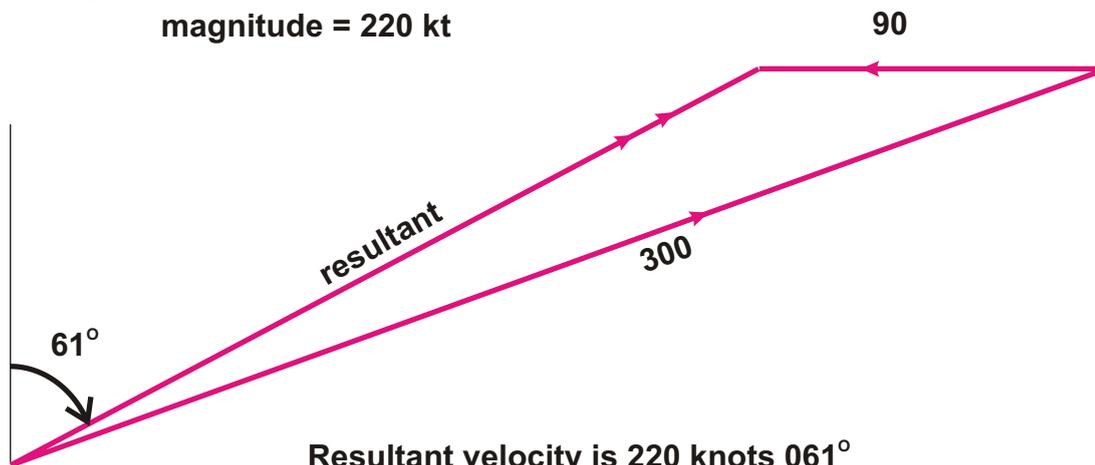
Method SCALE DRAWING

N

153°

200 m

100 m

resultant

500 m

Sketch to determine
layout and scale of
drawing.

Scale 4 cm = 100 N

Length of resultant = 17.8 cm

magnitude = 445 m

Resultant is 445 m 153° E of N

Example 2   *An aircraft is flying at 300 knots airspeed on a heading 070°. The air it is flying through has a velocity of 90 knots in a direction 240°. Find the resultant velocity of the aircraft relative to the ground.*

N
000

270 ——|—— 090

180

300 kt

90 kt

90 kt

resultant

300 kt
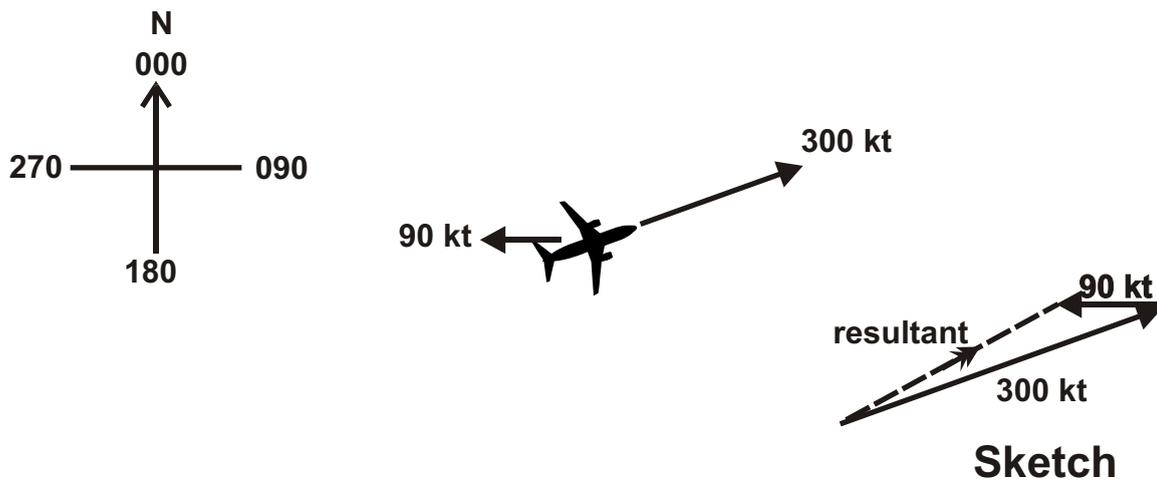
Sketch

Scale 5 cm = 100 kt

length of resultant = 11 cm

magnitude = 220 kt

90

resultant

300

61°

Resultant velocity is 220 knots 061°

Points to Note :

1. *Draw a direction marker (North)*

2. *Draw a sketch to work out how the scale drawing will look.*

3. *Choose a scale to make best use of the available paper. Note the scale.*

4. *Draw using a sharp pencil.*

5. *Resultant should have magnitude and a direction.*

Rectangular Components of a vector.

A number of vectors can be replaced by a single resultant vector. This simplifies the analysis of the effect of vectors on objects.
The reverse is true, we can replace a single vector by a number of component vectors. The sum of the component vectors will have a resultant equal to the original vector. This may seem an unnecessary complication, but if the component vectors are chosen to be at right angles, it allows the use of coordinate geometry and does away with the need for inaccurate scale drawing.
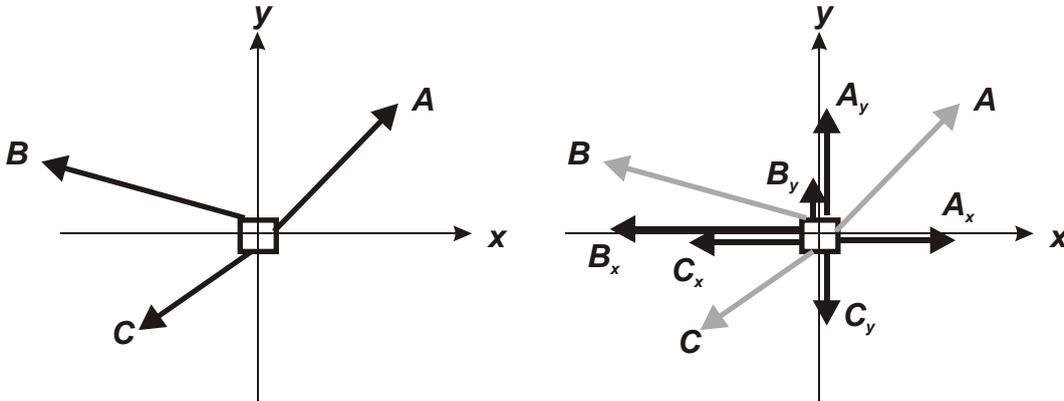
Vectors replaced by rectangular components
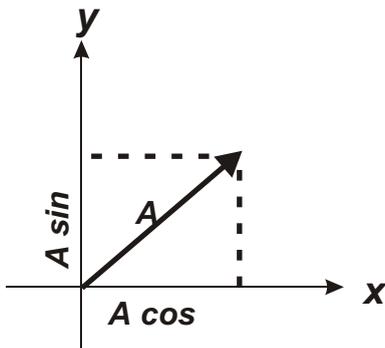
The process of replacing a vector by its rectangular components is known as **resolution**. Vectors are resolved into rectangular components.
Once the vector has been resolved, the original vector is replaced. We deal only with the components.

Example : *An object is projected with a velocity of 60 m.s^{-1} at an angle of 60° to the horizontal. Calculate the vertical and horizontal components of the velocity.*
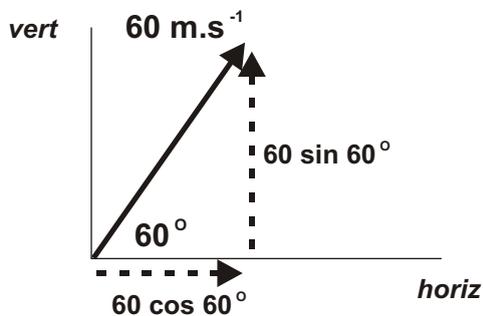
vertical component = 60 sin 60°
$\qquad\qquad\qquad$ = 52 m.s^{-1}

Horizontal component = 60 cos 60°
$\qquad\qquad\qquad$ = 30 m.s^{-1}

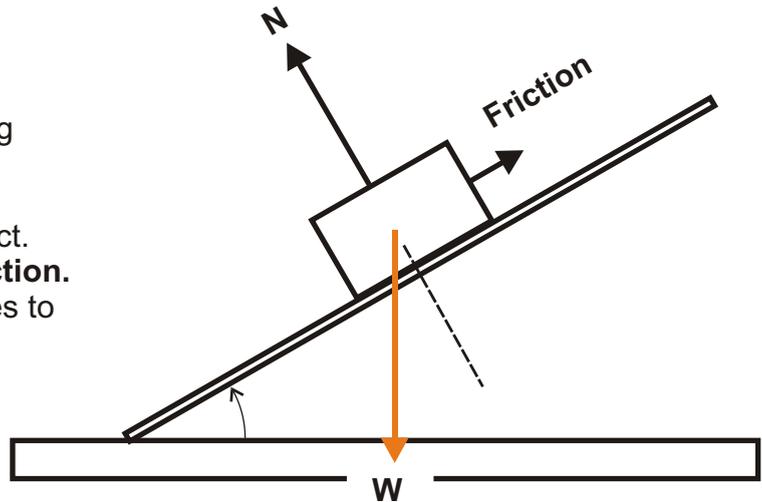## Inclined Plane.

Problems involving inclined planes
are common.
The forces acting on an object, moving
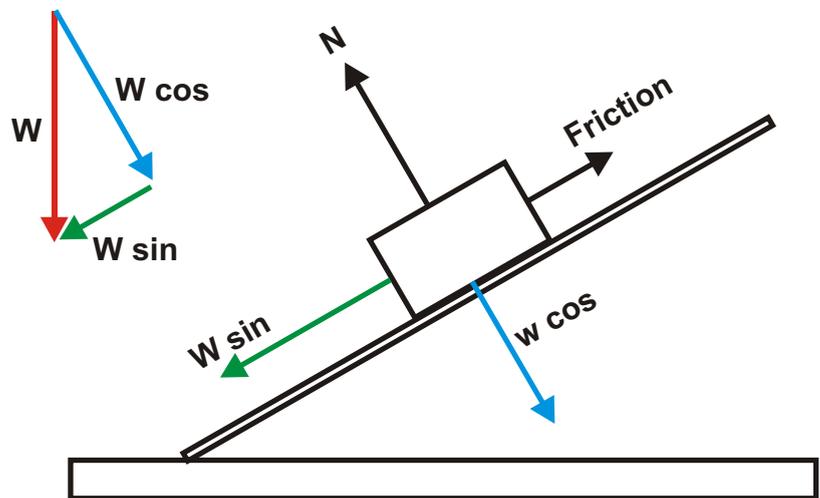down an inclined plane, are shown in
Fig. 1.
N is the force of the plane on the object.
This force is termed the **Normal Reaction.**
N is always found acting at right angles to
a surface

To solve this problem we resolve
the weight **W** into two rectangular
components. One acting down
the plane, the other at right
angles to the plane as shown.
The problem then reduces to

*Unbalanced force F, acting
down the plane*

$$F = W \sin \quad - Friction$$

Also   N = W cos

Equations of motion

Measurement of acceleration

Acceleration (*a*) : Acceleration is the change in velocity per unit time. Acceleration is a vector quantity measured in metres per second per second (ms^{-2}).

Measurement of acceleration : To measure the acceleration of an object we require to measure the velocity of the object at two points in its journey and the time taken to travel between the two chosen points.

Single obstructor and two light gates

Light gate 1 provides time for initial velocity.
Light gate 2 provides time for final velocity.
Time between light gates is recorded manually or electronically.

$$acceleration = \frac{change\ in\ velocity}{time}$$

timer

light gate 1

timer
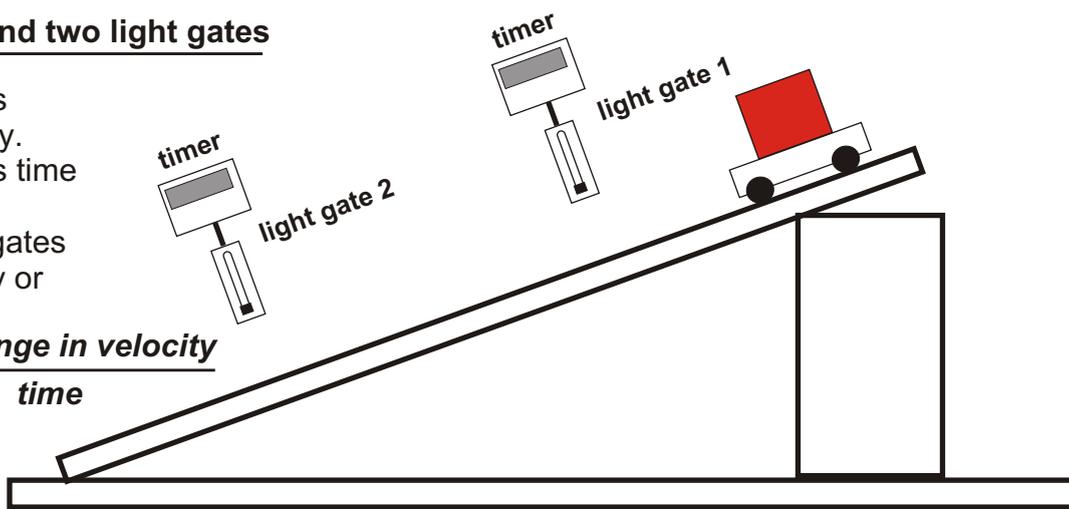
light gate 2

Double obstructor and single light gate

The motion computer measures the time taken for the first segment of card to pass the light gate and the time taken for the second segment. It also measures the time for the gap in between.
From this information and the length of the segment, the computer works out the acceleration.
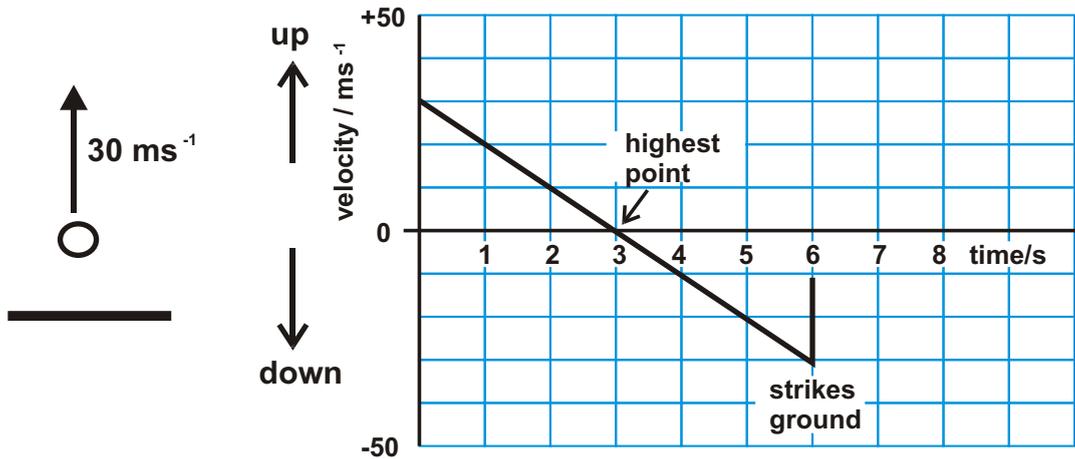
motion computer

light gate
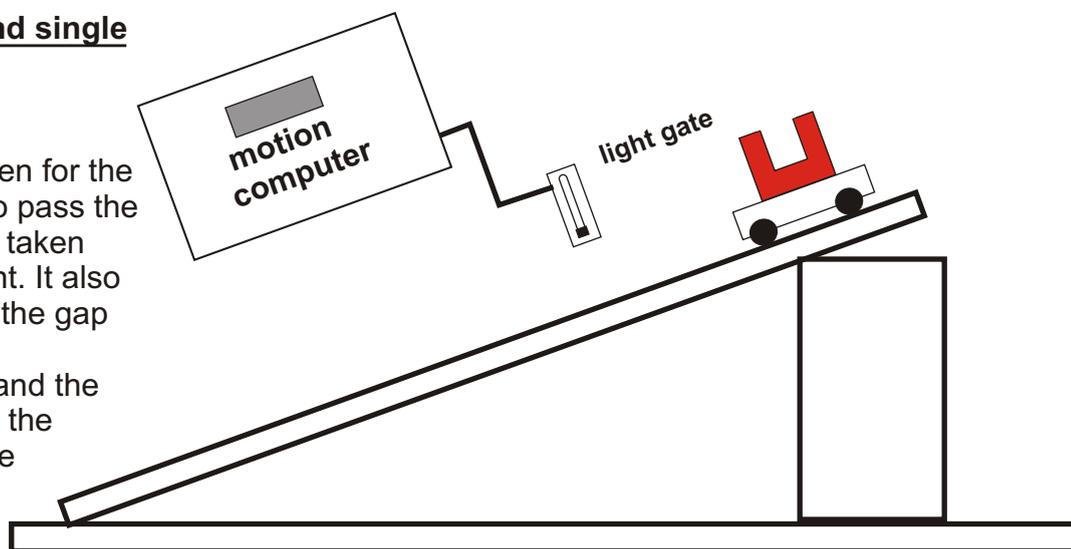
Velocity / acceleration - time graphs

A velocity - time graph shows the velocity of an object during a journey. As we are only dealing with linear motion, direction will be indicated by '+' and '-' signs.
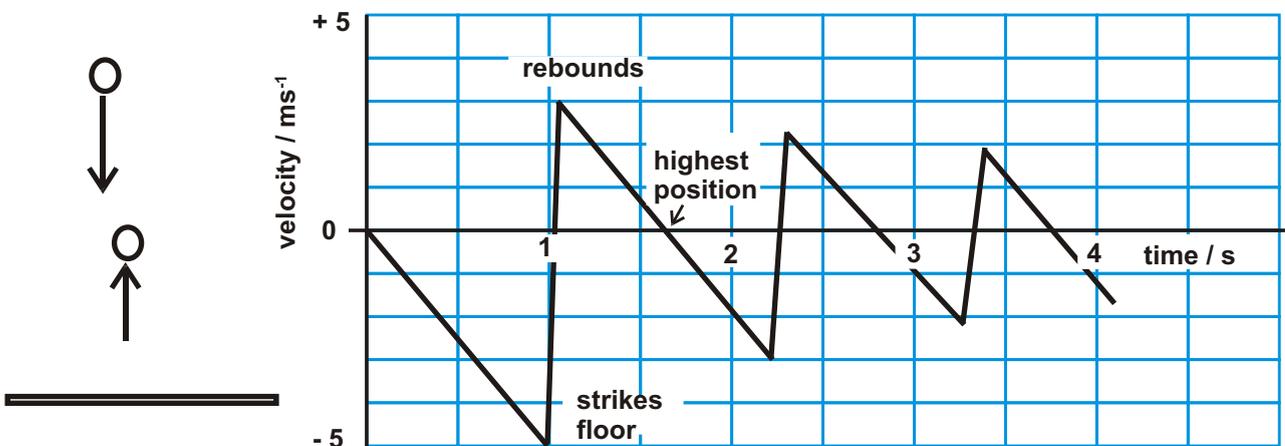
+ = upwards

- = downwards

- = to left

+ = to right
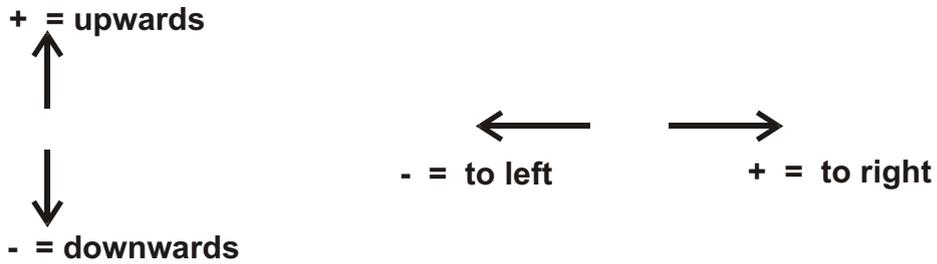
SIGN CONVENTION FOR VELOCITY

## Object projected vertically upwards

Bouncing ball

Acceleration - time graphs from velocity - time graphs.

The motion of an object can be described using a displacement - time graph, a velocity - time graph or an acceleration - time graph.

If we have one of these graphs we can derive the other two.

For Higher work you will be expected to derive an acceleration - time graph from a velocity - time graph. An example is shown below.
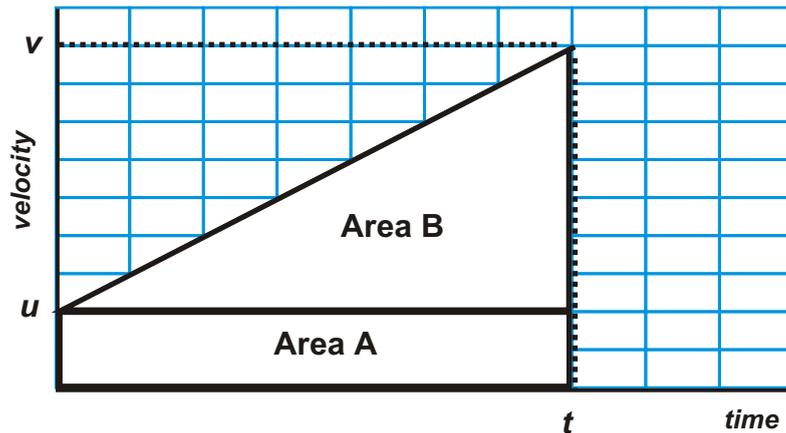

constant
acceleration
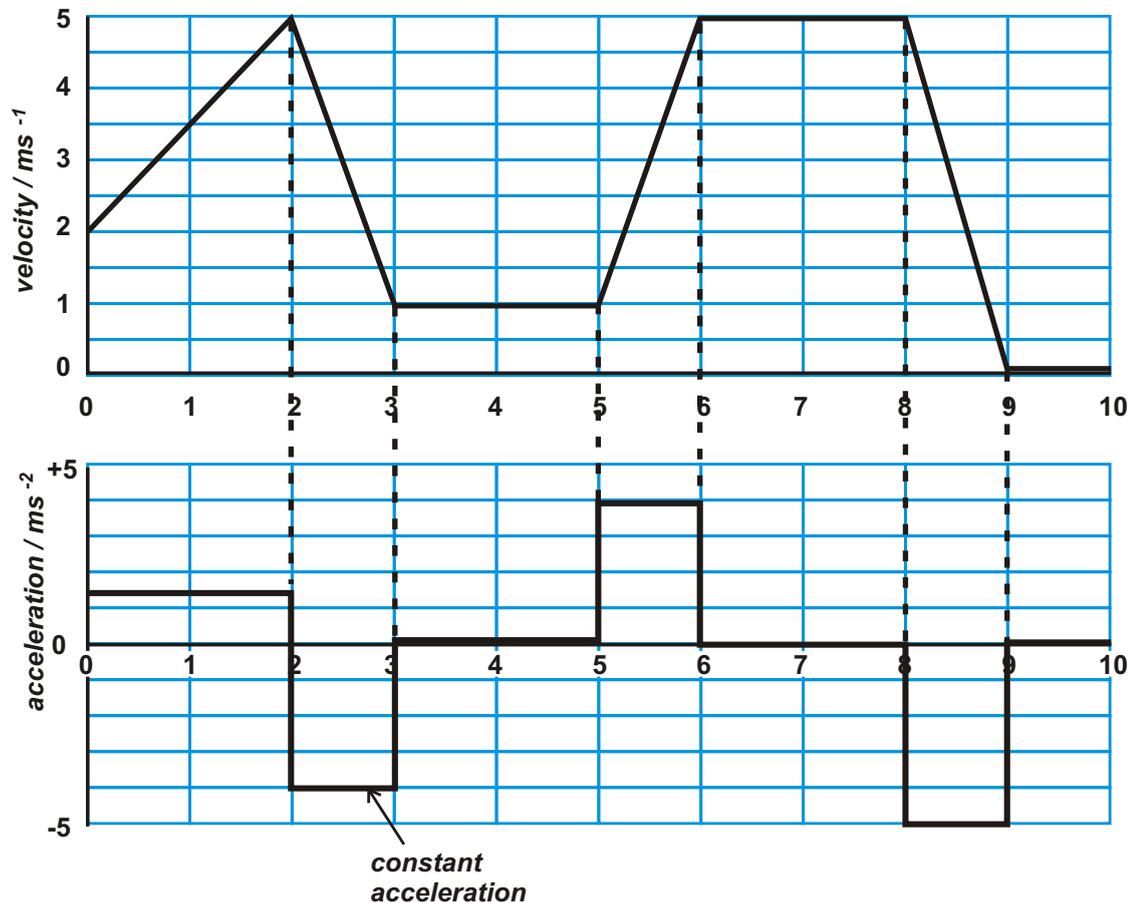
Equations of motion

The equations of motion are used to describe motion in a straight line involving *uniform acceleration*. There are three equations.

Equation 1 :

$$a \ = \ \frac{v - u}{t}$$

$$\boxed{v = u \ + \ at}$$

Equation 2 :

$$
\begin{aligned}
\text{displacement, } s \ &= \ \text{area under velocity - time graph} \\
&= \ \text{area A} \ + \ \text{area B} \\
&= \ ut \ + \ \tfrac{1}{2} \, t \, (v - u) \\
&= \ ut \ + \ \tfrac{1}{2} \, t \, .at \ .... \ (v - u) = at \qquad \text{from Eq. 1} \\
&= \ ut \ + \ \tfrac{1}{2} \, at^2
\end{aligned}
$$

$$\boxed{s \ = \ ut \ + \ \tfrac{1}{2} \, at^2}$$

Equation 3 :

$$\text{substitute } t \ = \ \frac{v - u}{a} \quad \text{in Eq. 2}$$

$$s \ = \ u \, \frac{(v - u)}{a} \ + \ \tfrac{1}{2} \, a \, \frac{(v - u)^2}{a^2}$$

$$s \ = \ u \, \frac{(v - u)}{a} \ + \ \frac{(v - u)^2}{2a}$$
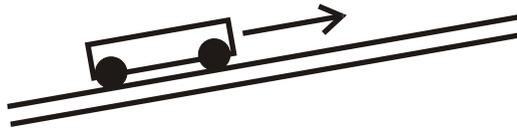
multiply both sides by 2a

$$
\begin{aligned}
2as \ &= \ 2u(v - u) \ + \ (v - u)^2 \\
&= \ 2uv - 2u^2 + v^2 - 2uv \ + \ u^2 \\
&= \ v^2 \ - \ u^2
\end{aligned}
$$

rearranging

$$\boxed{v^2 \ = \ u^2 \ + \ 2as}$$

Example: A trolley is projected up a slope with a velocity of 12 ms^{-1}. The acceleration on the trolley is 3 ms^{-2} acting down the slope.

Find **(a)** velocity of the trolley after 6 s.

(b) the displacement of the trolley after 6 seconds.

(a)

$v = u + at$

$v = 12 - 3 \times 6$

$= -6 \ ms^{-1}$

$u = 12 \ ms^{-1}$ up the slope
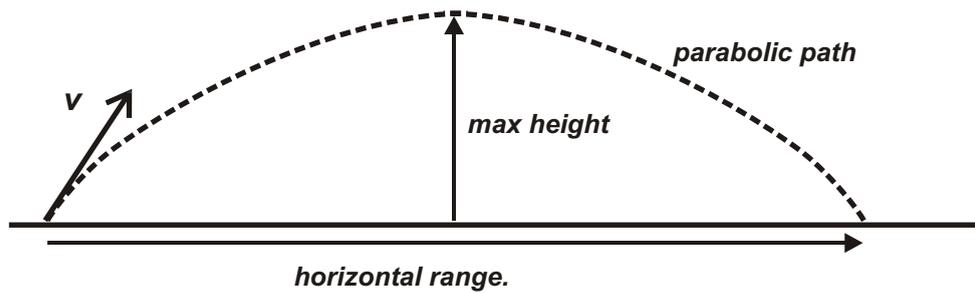
$a = -3 \ ms^{-2}$ (down the slope)

$t = 6s$

velocity after 6 s is 6 ms^{-1} down the slope.

(b)

$s = ut + \frac{1}{2}at^{2}$

$= 12 \times 6 - 0.5 \times 3 \times 6 \times 6$

$= 18 \ m$

displacement after 6 s is 18 m up the slope.

Projectile motion

Projectile motion is motion in two dimensions. this problem is dealt with by converting to two separate motions in one dimension : resolving into horizontal and vertical components and dealing with each component independently.

vertical component $v_v = v\ sin$

horizontal component $v_H = v\ cos$
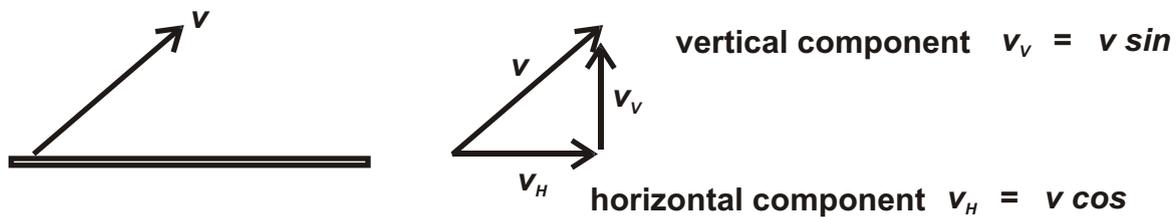
The horizontal component is not affected by gravity and remains constant throughout the flight.
The vertical component is affected by gravity.

Example: An object is projected with a velocity of 50 ms^{-1} at an angle of 20° to the horizontal.
Find **(a)** The maximum height reached;
        (b) the horizontal range.

Find horizontal and vertical components of initial velocity.

$$horizontal\ =\ 50\ cos\ 20\ =\ 47\ ms^{-1}$$
$$vertical\ =\ 50\ sin\ 20\ =\ 17\ ms^{-1}$$

(a)

Maximum height reached when vertical component = 0

$v = u + at$ $v = 0$

$0 = 17 - 9.8t$

$t = 1.73\ s$ $u = 17\ ms^{-1}$

                        $a = -9.8\ ms^{-2}$

time to maximum height = 1.73 s

continued ....

(a) continued

$$s = ut + \tfrac{1}{2} at^2$$

$$= 17 \times 1.73 - 0.5 \times 9.8 \times 1.73^2$$

$$= 29.4 - 14.7$$

$$= 14.7 \text{ m}$$

**maximum height = 14.7 m**

(b) maximum height is reached at mid point of journey so that the time taken for the complete flight is twice the time taken to reach the maximum height.

**The horizontal component of velocity remains constant**

$$\text{horizontal range } s = v_H \times t$$

$$= 47 \times 2 \times 1.73$$

$$= 162.6 \text{ m}$$

**horizontal range = 162.6 m**

If there was no requirement to find the time of flight we could have used equation of motion no. 3 to find the maximum height.

$$v^2 = u^2 + 2as$$

$$0 = 17^2 - 2 \times 9.8 \times s$$

$$s = \frac{289}{2 \times 9.8}$$

$$= 14.7 \text{ m} \quad \text{the same answer as before.}$$

Newton's Second Law, energy and power.

Newtons Second Law.

Newtons' Second Law deals with the effect of forces on moving objects, and provides a means to define the unit of force.

$$F = m.a$$

resultant force — mass — acceleration

If the mass is measured in kg, and the acceleration is measured in ms^{-2}, then force is measured in Newtons (N)

1 Newton is the resultant force which will accelerate an object with mass 1 kg at a rate of 1 ms^{-2}.

Free body diagrams.

A free body diagram is a diagram which shows all the forces acting on an object. Free body diagrams should be drawn before attempting to apply Newtons' Second Law to any situation.

Example.

A rocket has a mass of 10000 kg at lift off and develops a thrust of 150000 N. Calculate its acceleration at lift off.
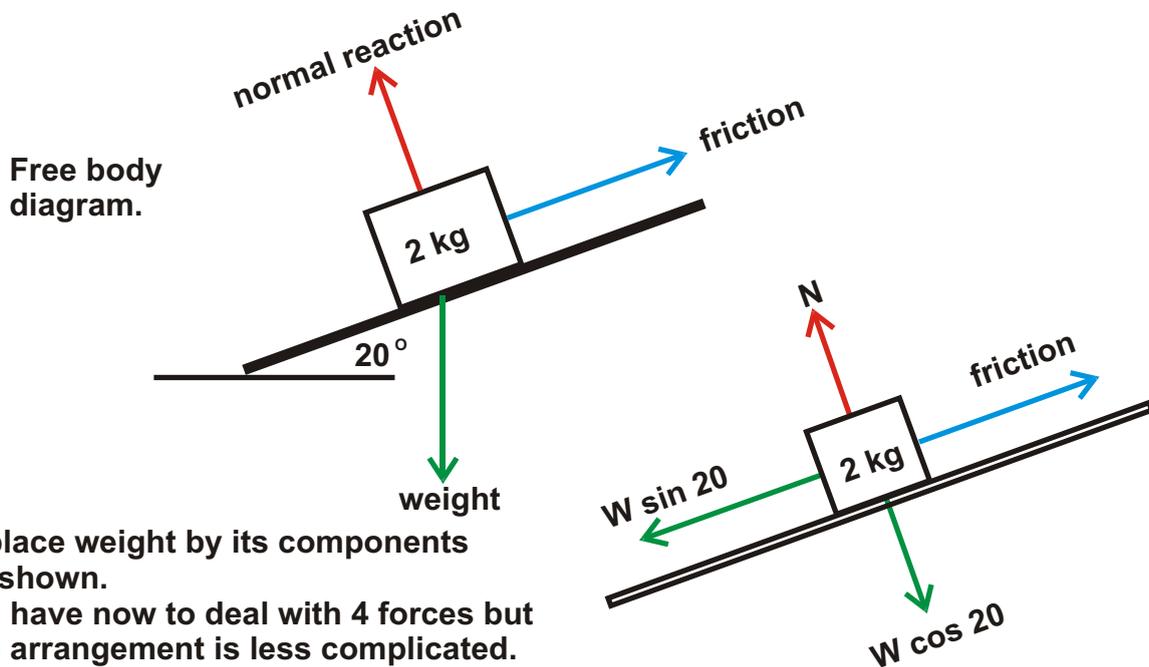
F = *thrust - weight*
   = *150000 - 10000 x 9.8*
   = *52000 N*

$F = m.a$

$52000 = 10000 \times a$

$a = 5.2\ ms^{-2}$

thrust

10000 kg

weight

free body diagram
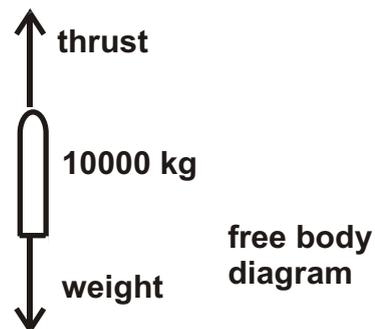
acceleration at take off is 5.2 ms^{-2}

Example. A block, mass 2 kg, slides down a slope which is inclined at an angle of 20° to the horizontal. The block has a constant acceleration of 1.5 ms⁻² down the slope. Calculate the force of friction on the sliding block.

Free body diagram.

normal reaction

friction

2 kg

20°

weight

replace weight by its components as shown.
We have now to deal with 4 forces but the arrangement is less complicated.

N

friction

2 kg

W sin 20

W cos 20

The block will move in the direction of the resultant force. This means the resultant force acts down the slope. The two forces acting at right angles to the slope have no effect so:

N + W cos 20 = 0

Both forces have the same magnitude but are opposite in direction.

So the Resultant force, F = w sin 20 - friction

m.g sin 20 - friction = m .a

2 x 9.8 sin 20 - friction = 2 x 1.5

6.7 - friction = 3.0

friction = 4.7 N

Friction acting on the sliding block = 4.7 N

Work Done.

Work is done on an object when a force is used to move the object. The Work Done is given by:
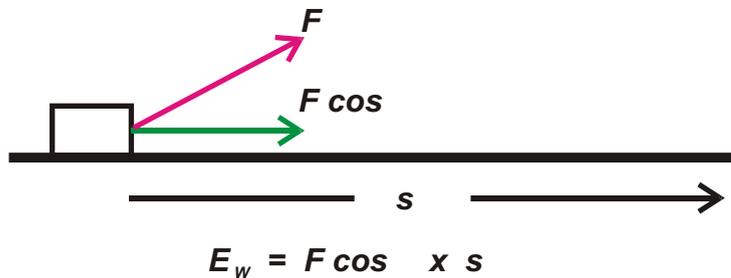
work done = force applied x distance moved.

$$E_w = F \; x \; s$$

If the force is in a different direction from the movement, then the work done is given by:

work done = component of force x distance moved.

$$E_w = F \cos \; x \; s$$

Work Done is a scalar quantity measured in Newton metres (Nm)

When work is done on an object, energy is transferred to the object (and the surroundings if friction is involved). The quantity of energy transferred is equal to the work done.
One Joule of energy is transferred for each Newton metre of work done.
The quantity of energy in a source is defined as its potential to do work. To measure the energy we need to convert all the energy to work done and measure the work done.

In mechanical systems, work done on an object transfers kinetic and potential energy to the object and frictional heat to the surroundings. The quantity of energy transferred to heat is equal to the work done overcoming the force of friction.

work done = kinetic energy gained + potential energy gained + heat
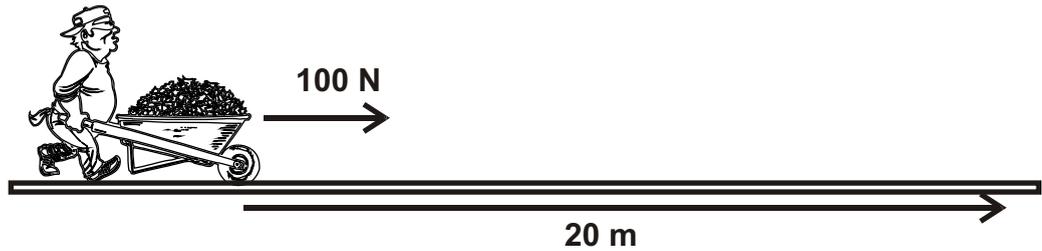
heat = force of friction x distance moved.

$$F \; x \; s \; = \; \tfrac{1}{2} \, m v^2 \; + \; mgh \; + \; F_{frict} \, x \; s \quad typically.$$

Energy can also be transferred to work done.

The Conservation of Energy applies to situations involving work done. The work done on an object must always equal the energy transferred to the object.

Example.

100 N

20 m

A workman pushed a wheelbarrow from rest using a steady force of 100 N. He moved the wheelbarrow a distance of 20 m reaching a speed of 4 ms^{-1}.
Find the average force of friction acting on the wheelbarrow.

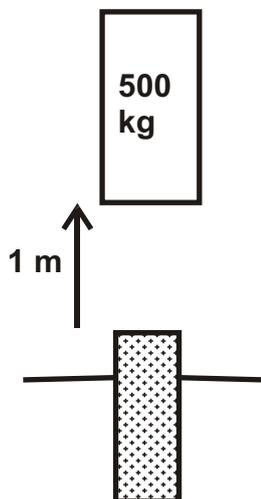*Work done = kinetic energy gained + work done overcoming friction.*

$$F\,s \ = \ \tfrac{1}{2}\,mv^{2} \ + \ F_{frict} \times s$$

$$100 \times 20 \ = \ 0.5 \times 50 \times 16 \ + \ F \times 20$$

$$2000 \ = \ 400 \ + \ F \times 20$$

$$F \ = \ 80 \ N$$

Force of friction = 80 N

500 kg

1 m

A 500 kg mass is dropped a distance of 1 m to strike the top of a concrete pile.
The pile is driven in a distance of 20 cm. Find the average driving force on the pile.

Energy lost = work done driving pile.

$$mgh \ = \ F.s$$

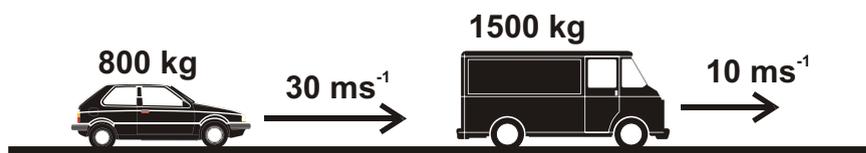$$500 \times 9.8 \times 1.2 \ = \ F \times 0.2$$

$$F \ = \ 29400 \ N$$

note: potential energy lost by falling mass is given by:

$$E = 500 \times 9.8 \times 1.2$$

The mass dropped a further 20 cm driving the pile

Average force = 29400 N

Momentum and impulse

Momentum

The momentum of a moving object is the product of its mass and velocity. Momentum is a vector quantity measured in kg ms^{-1}.

$$momentum = mass \times velocity$$
$$p = m.v$$

3 ms^{-1} $\longrightarrow$ 20 kg   3 ms^{-1} $\longleftarrow$ 20 kg

momentum = + 60 kg ms^{-1} momentum = - 60 kg ms^{-1}

Any process which changes the speed of an object, will change its momentum.
A resultant force will cause a change in velocity (**F = ma**). The change in momentum caused by a resultant force is called an **Impulse.**

Collisions

When two objects collide, the collision generates forces which act on both objects. As a result of this, the momentum of both objects is changed by the collision.
If there are no external forces acting on the objects and the only forces are those created by the collision, the vector sum of the momentums before the collision is equal the vector sum of the momentums after the collision. We say that momentum is conserved.

Conservation of Momentum .   When two objects collide, the vector sum of the momentums before the collision is equal to the vector sum of the momentums after the collision provided there are no external forces involved.

U_A (A) $\longrightarrow$ U_B $\longleftarrow$ (B)   *before the collision*

V_A $\longleftarrow$ (A)   (B) V_B $\longrightarrow$ *after the collision*

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B \quad vector \; addition$$

External forces which might be involved include gravity, friction and electric forces.

Example:

800 kg   30 ms⁻¹   1500 kg   10 ms⁻¹

A car, mass 800 kg, travelling at a velocity of 30 ms⁻¹, collides with a van, mass 1500 kg, travelling in the same direction with a velocity of 10 ms⁻¹. After the collision both vehicles are locked together.
Calculate the speed of the two vehicles immediately after the collision.

Sum of the momentums before the collision.

$$800 \times 30 \ + \ 1500 \times 10 \ = \ 39000 \ kgms^{-1}$$
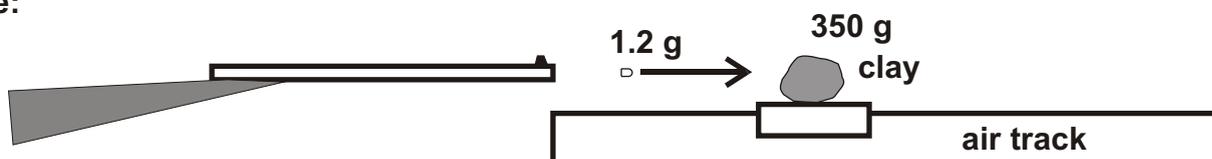
Sum of the momentums after the collision

$$= \ (800 + 1500) \times v \ kg \ ms^{-1}$$

Applying the conservation of momentum.

$$(800 + 1500) \times v \ = \ 39000$$

$$v \ = \ 17 \ ms^{-1} \ (\ rounding \ up \ )$$

Example:

1.2 g   350 g clay   air track

An air gun pellet, mass 1.2 g, is fired horizontally to strike a piece of modelling clay mounted on a stationary air track vehicle. After the pellet strikes the clay, the vehicle moves off with a velocity of 0.67 ms⁻¹. The total mass of the clay and vehicle was 350 g.
Calculate the speed of the air gun pellet.

Sum of the momentums before the collision

$$1.2 \times 10^{-3} \times v \ + \ 350 \times 10^{-3} \times 0 \ = \ 1.2 \times 10^{-3} \times v \ kg \ ms^{-1}$$

Sum of the momentums after the collision

$$= \ (1.2 \times 10^{-3} \ + \ 350 \times 10^{-3}) \times 0.67 \ kg \ ms^{-1}$$

Apply the conservation of momentum

$$(1.2 \times 10^{-3} \ + \ 350 \times 10^{-3}) \times 0.67 \ = \ 1.2 \times 10^{-3} \times v$$

$$v \ = \ 196 \ ms^{-1}$$

Explosions.

When an object explodes into different fragments, the only forces involved are inside the object. With no external forces, the *Conservation of Momentum* applies. The sum of the momentums of the fragments is equal to the original momentum of the object.
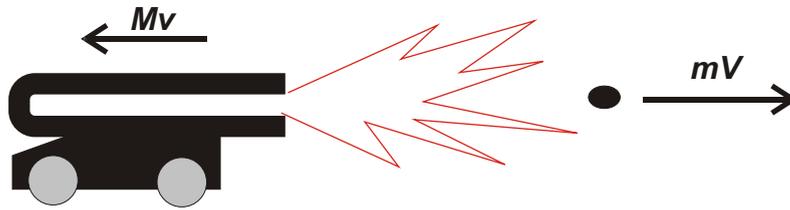
Recoil of a gun when fired

$$mV - Mv = 0 \ (\text{original momentum of gun})$$
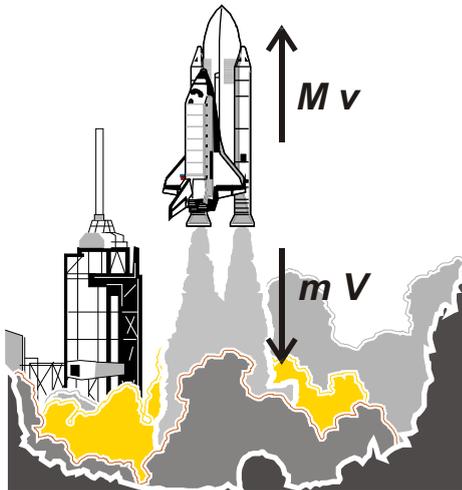
Rocket engines create a continuous explosion. The explosion creates the momentum of the exhaust gases and the momentum of the rocket in the opposite direction.

Elastic and inelastic collisions

An elastic collision is one where there is no loss of **Kinetic Energy** during the collision. Elastic collisions take place between the particles in a gas and their container, and between charged or magnetised objects where no contact occurs.
Kinetic energy is lost during an inelastic collision. Some kinetic energy is usually converted to heat or work done bending the colliding objects.
Total Energy is always conserved during collisions.

Example
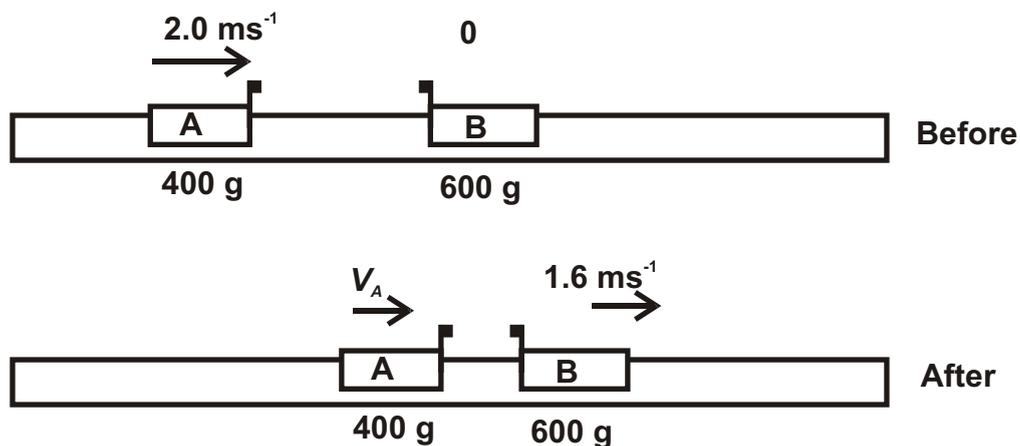
During an experiment, a vehicle, mass 400 g, is pushed so that it runs along an air track with a velocity of 2.0 ms^{-1}. It collides with a stationary vehicle, mass 600 g, which moves on with a velocity of 1.6 ms^{-1}.
(a) Find the velocity of the 400 g vehicle after the collision.
(b) Show that this was an elastic collision.

(a) *Sum of momentums before the collision = 0.4 x 2.0 + 0.6 x 0 = 0.8 kg ms^{-1}*

Sum of the momentums after the collision = 0.4 x V$_A$ + 0.6 x 1.6 kg ms^{-1}

Apply the conservation of momentum

$$0.4 \text{ x } V_A + 0.6 \text{ x } 1.6 = 0.8$$
$$0.4 \text{ x } V_A = 0.8 - 0.96$$
$$= -0.16$$
$$V_A = -0.4 \text{ ms}^{-1}$$

Velocity of 400 g vehicle after the collision is 0.4 ms^{-1} to the left.

(b)
Kinetic energy before the collision = 0.5 x 0.4 x (2.0)^2 = 0.8 J

Kinetic energy after the collision = 0.5 x 0.4 x (0.4)^2 + 0.5 x 0.6 x (1.6)^2

$$= 0.032 + 0.768$$
$$= 0.8 \text{ J}$$

No kinetic energy has been lost during the collision so collision is elastic.

Newtons' Third Law
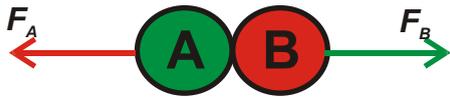
$$F_A = -F_B$$

During a collision between two objects A and B, the force on A due to B is equal and in the opposite direction to the force on B due to A.

This is a direct result of the conservation of momentum. The change in momentum experienced by A is equal and in the opposite direction to the change in momentum experienced by B.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

rewritten
$$m_A(v_A - u_A) = -m_B(v_B - u_B)$$

$$\text{change in momentum A} = -\text{change in momentum B}$$

If objects are in contact for time t seconds.

dividing by t
$$\frac{m_A(v_A - u_A)}{t} = -\frac{m_B(v_B - u_B)}{t}$$

$$m_A a_A = -m_B a_B$$

$$F_A = -F_B$$
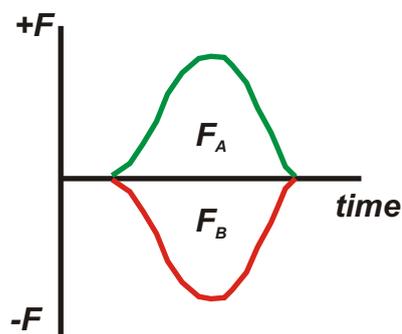
The force acting on A is mirrored by the force acting on B. They are equal and opposite at any time during the collision.

Impulse

A constant force **F**, acting on mass **m**, over a time **t**, will cause the mass to accelerate from velocity **u** to velocity **v** such that.

$$F = m \frac{(v - u)}{t}$$

If we rewrite this:

$$Ft = mv - mu = \textit{change in the momentum of the object.}$$

Impulse is the product of force and the time over which the force acts. Impulse is measured in Newton seconds (Ns)

Impulse (J) = Force x time for a constant or average force.

$$J = F t$$

Impulse = change in momentum of object affected by impulse.

Where the force is not constant.

Impulse = area under F - t graph
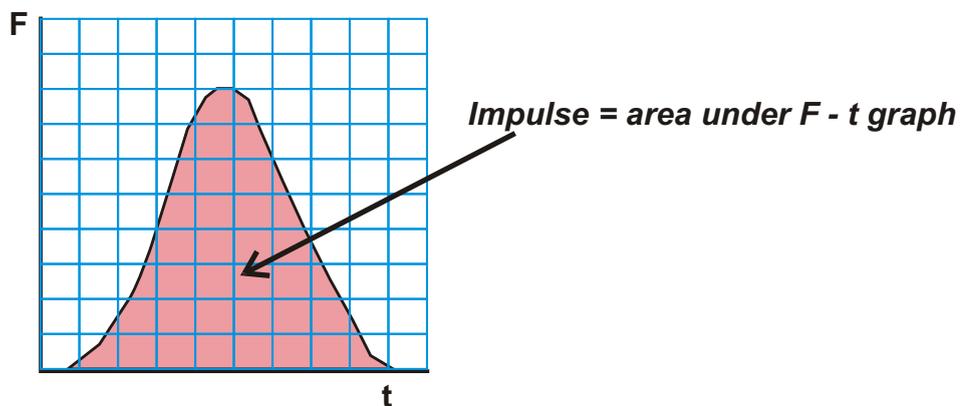
Measurement of Average Force
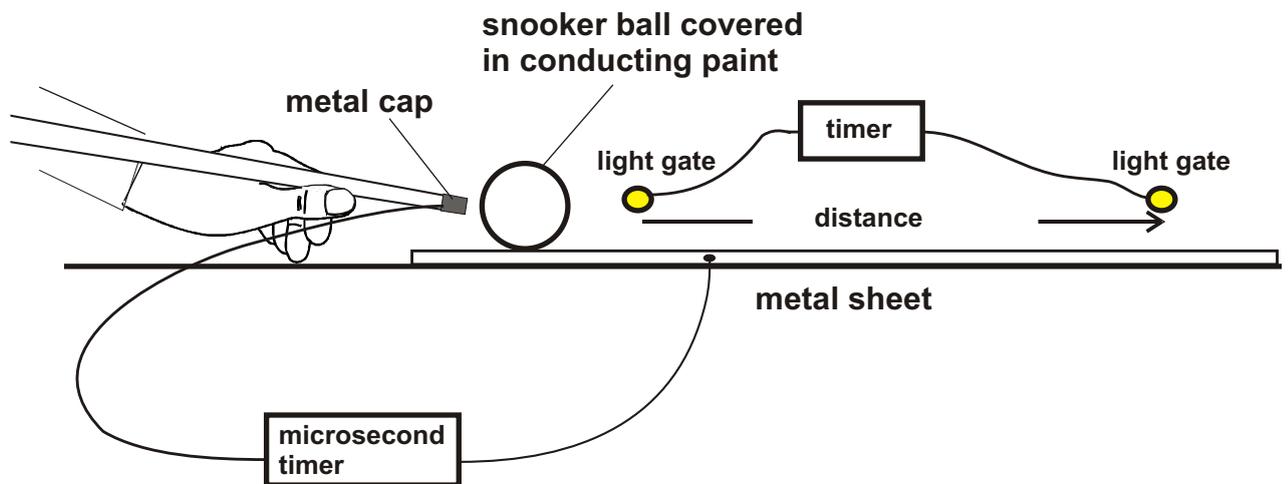

```
Mass of ball            =   160 g

Distance between lightgates  =  0.4 m

Time between lightgates  =  0.56 s

Contact time            =   25 ms
```

Momentum gained by ball   =   mass x velocity

$$= \ 0.16 \ \times \ \frac{0.4}{0.56}$$

$$= \ 0.114 \ kg \ ms^{-1}$$

Impulse = change in momentum

$F\,t \ = \ 0.114$

$F \times 25 \times 10^{-3} \ = \ 0.114$

$F \ = \ 4.6 \ N$

Average force on ball = 4.6 N

Modifying Impact

The forces involved during collisions can be extremely damaging. Objects which may be involved in collisions need to be designed to cope with the situation.
The way this is done is to try and extend the contact time between the colliding objects. This has the effect of decreasing the magnitude of the forces involved.

For example a car, mass 1200 kg , travelling at 13 ms^{-1}, is brought to rest in 1 second during a collision. Calculate the average force acting on the car during the collision.

$$F\,t\ =\ \textit{change in momentum}$$

$$F \times 1\ =\ 1200 \times 13$$

$$F\ =\ 15600\ N$$

The average force acting on the car is 15600 Newtons. The maximum force will be at least double this value.

A modern car is designed to crumple progressively, extending the contact time to 3, 4 or more seconds. This reduces the average and maximum forces acting. The force of the seatbelt on both passengers and driver is also reduced as the car takes longer to come to rest.

If the collision time was increased to 4 seconds

$$F\,t\ =\ \textit{change in momentum}$$

$$F \times 4\ =\ 1200 \times 13$$

$$F\ =\ 3900\ N$$

The force acting on a passenger, mass 70 kg, would be reduced from 970 Newtons to a safer 243 Newtons. This only works if the passenger slows down at the same rate as the car. A seat-belt is worn!!

Pressure and density

Density

The density of a substance is the mass per unit volume of that substance. Density is measured in kilograms per cubic metre (**kg m^{-3}**).

$$density \ = \ \frac{mass}{volume}$$

$$= \ \frac{m}{V}$$

Greek letter 'ro'

Example: Find the density of mercury, if 200 cm^3 of mercury has a mass of 2.72 kg

$$= \ \frac{m}{V}$$

$$= \ \frac{2.72}{200 \ x \ 10^{-6}}$$

$$= \ \underline{13600 \ kg \ m^{-3}}$$

$V \ = \ 200 \ cm^3$

$\quad = \ 200 \ x \ 10^{-6} \ m^3$

$m \ = \ 2.72 \ kg$

Measuring the density of Air

valve

rigid plastic bottle

tyre pump

measuring cylinder

water
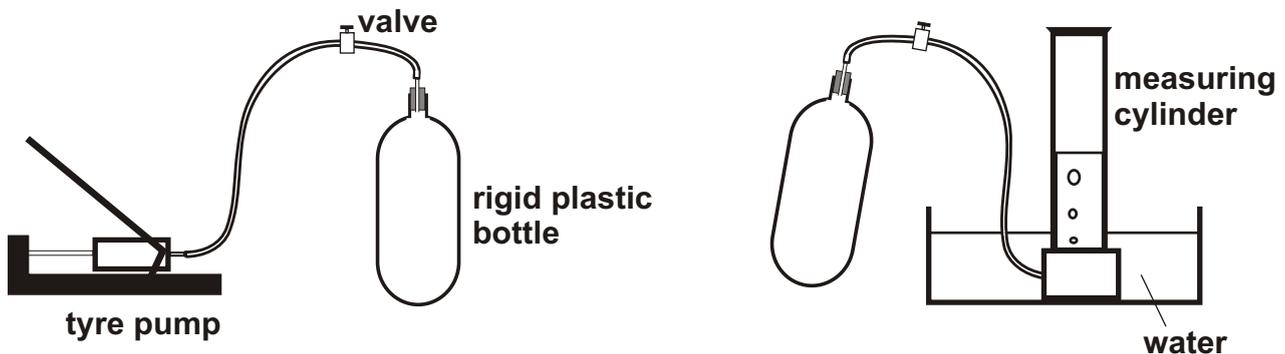
1. *Air is pumped into a RIGID container then valve is closed.*

2. *Container is detached from pump and its mass measured using a SENSITIVE balance.*

3. *The extra volume of air inside the container is measured by bubbling the air into a measuring cylinder as shown.*

4. *The mass of the container is measured again.*

$$density \ of \ air \ = \ \frac{mass \ of \ full \ container \ - \ mass \ of \ empty \ container}{measured \ volume \ of \ air}$$

Result = 1.2 kg m^{-3}

Solid, Liquid and Gas

When comparing the densities of the different phases of a substance, we find that the densities of the solid and liquid phases are roughly equal, but the density of the gas is 1000 times less than the liquid or gas.

The size of the particles in a substance are unaffected by the changes between solid, liquid and gas. The difference in density is due to the difference in the spacing between the particles. The particles in a gas are 10 times further apart than the particles in the liquid or solid.

solid / liquid

volume = 1 x 1 x 1

10

10

10

10

gas

volume = 10 x 10 x 10
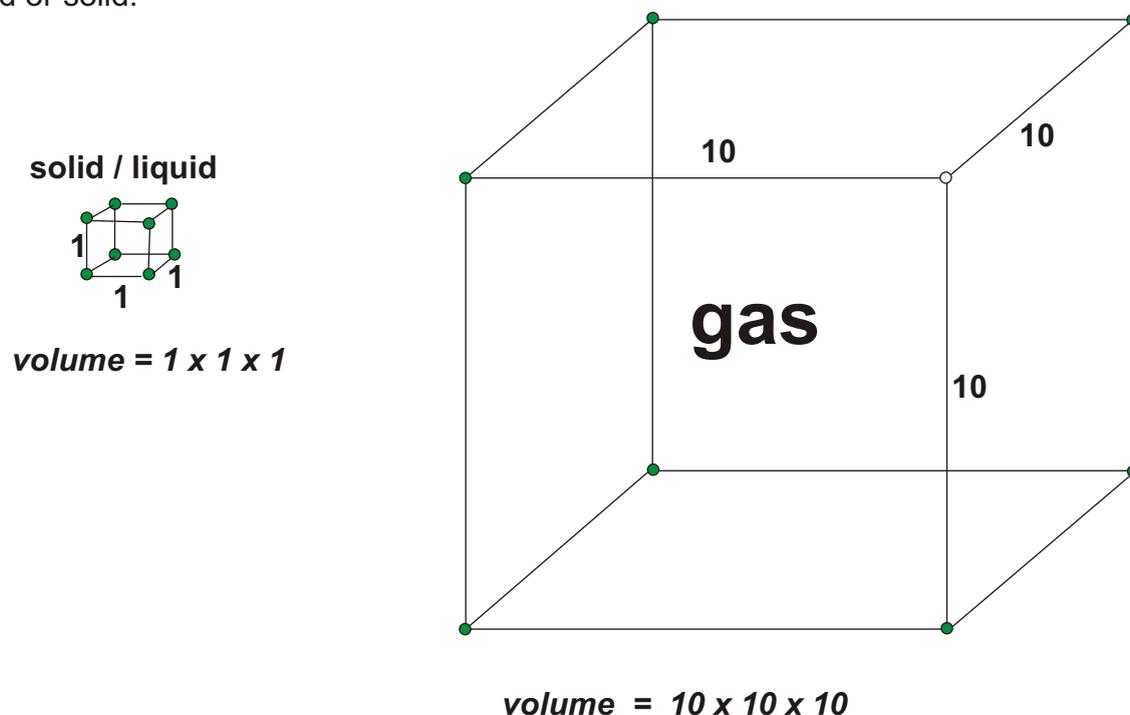
Solid carbon dioxide can be collected from a fire extinguisher and formed into a 1 cm cube. When it is held under a water filled measuring cylinder, as shown, the solid carbon dioxide quickly turns to a gas and is collected. Approximately 1000 cm^3 of gas is collected.

1000 cm^3 CO_2 gas

tongs

1 cm^3 solid CO_2
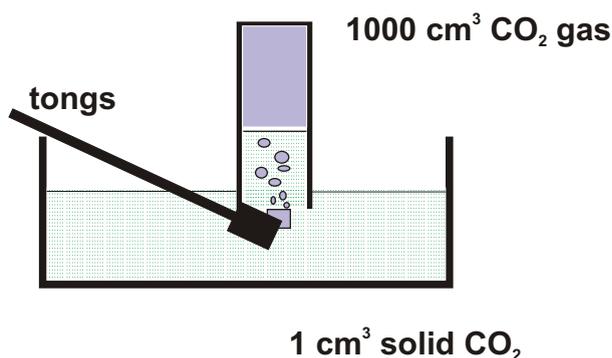
Pressure

Pressure is a measure of the effect of forces on surfaces. Pressure is the force per unit area, when the force acts normal to the surface.
If the force is not normal to the surface then the component of the force, normal to the surface is used.
Pressure is measured in pascals, where 1 pascal = 1 newton per square metre.

$$pressure = \frac{force}{Area}$$

$$p = \frac{F}{A} \qquad \text{Note : symbol for pressure is 'small' p}$$

Example:

A rectangular block, made from lead (density 11343 kg m^3) is resting on a horizontal surface as shown.
Calculate the pressure on the surface due to the block.

20 cm

15 cm

30 cm
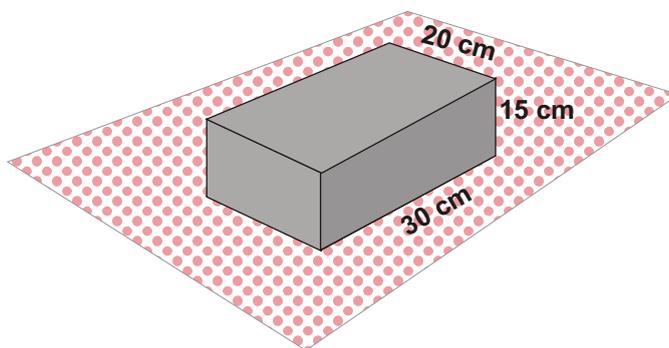
$$p = \frac{F}{A}$$

$$= \frac{1000.4}{0.06}$$

$$= 16673.3\ Pa$$

pressure = 16673.3 Pa

F = weight of block

$$= m\,g$$

$$= V\,g$$

$$= 11343 \times 20 \times 15 \times 30 \times 10^{-6} \times 9.8$$

$$= 1000.4\ N$$

$$A = 30 \times 20 \times 10^{-4}$$

$$= 0.06\ m^2$$

Calculate the pressure when the surface is inclined 30° to the horizontal.

$$p = \frac{F}{A}$$

$$= \frac{W\cos 30}{A} \qquad \text{normal component of W}$$

$$= \frac{1000.4 \times \cos 30}{0.06}$$

$$= 14439.5\ Pa$$

w cos 30

W

30°
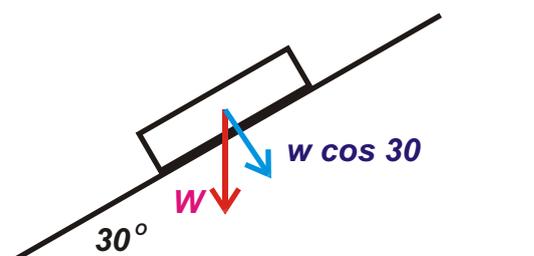
Pressure in Liquids

The pressure in a liquid or gas increases with depth.
The pressure at a point in a liquid at rest is given by the relationship;

pressure   = density x gravitational field strength x depth

$$p = \rho g h$$

This relationship does not apply to gases, as the density of a gas is not constant, but increases with depth.

Pressure at a point in a liquid or gas is created by particle collisions on a surface. The particles move in random motion which means the average speed of the particles in any direction is the same. This means that the pressure on the surface will be the same for any orientation of the surface. Pressure in a liquid or gas acts equally in all directions.
Pressure is a scalar quantity.

Upthrust.

An object, immersed in a liquid or a gas experiences a force which acts vertically upwards. This force, termed the upthrust on the object, is caused by the pressure on the deeper bottom of the object being greater than the pressure on the shallower top. Pressure increases with depth.

UPTHRUST
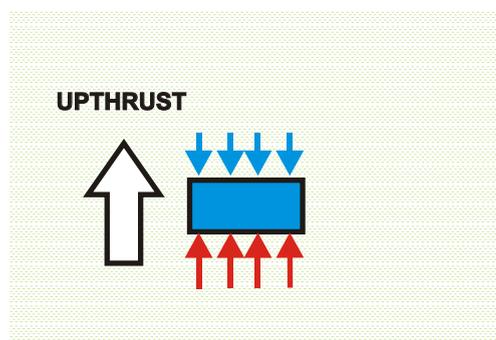
Example: Calculate the increase in pressure experienced by a submarine when it dives to a depth of 400 m under the sea.

$p = \rho g h$

$= 1030 \times 9.8 \times 400$

$= \underline{4.03 \times 10^6 \ Pa}$

sea water ρ = 1030 kg m^{-3}
g = 9.8 N kg^{-1}
h = 400 m

Pressure is approx 40 x atmospheric pressure.

Gas Laws

The Kinetic Model   Gases are composed of freely moving particles, moving around randomly a high speeds. The particles collide elastically with each other and the walls of their container. The only energy carried by the particles is kinetic energy which can only be changed by changing the temperature of the gas.

The Kinetic Model tries to explain the Gas Laws through the behaviour of the particles. The Gas Laws define the relationship between the pressure, volume and temperature of a fixed mass of gas.

Boyles' Law
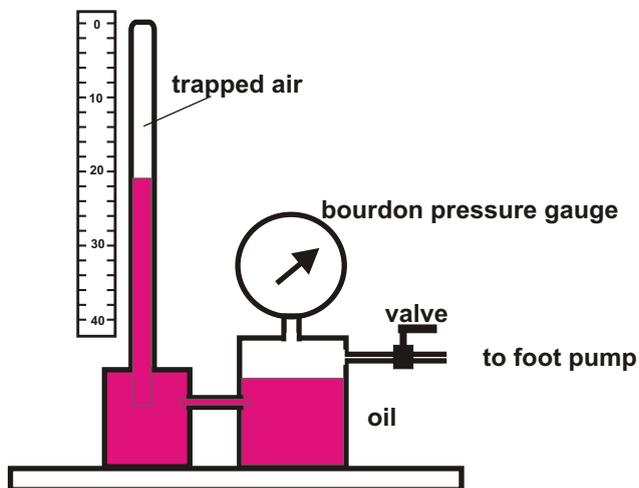
Boyles' Law apparatus

The pressure in a fixed mass of gas at constant temperature varies inversely as the volume of the gas.

$$p \propto \frac{1}{V}$$

$$p_1 V_1 = p_2 V_2$$
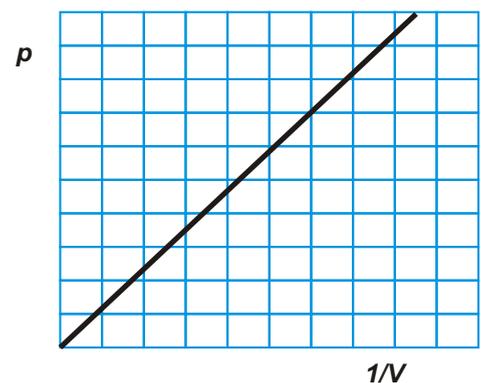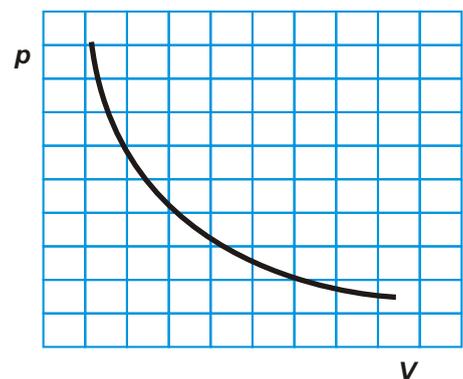
Charles' Law

The volume of a fixed mass of gas at constant pressure varies directly as its temperature measured on the kelvin scale

$$T(K) = T(^oC) + 273$$

$$V \propto T$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

capillary tube

ruler

thermometer

mercury

trapped air

heat SLOWLY

volume - temperature (^oC)

volume

-300   **-200**   **-100**   **0**   **+100**

temperature / ^oC

volume - temperature (K)

volume

0   **100**   **200**   **300**   **400**
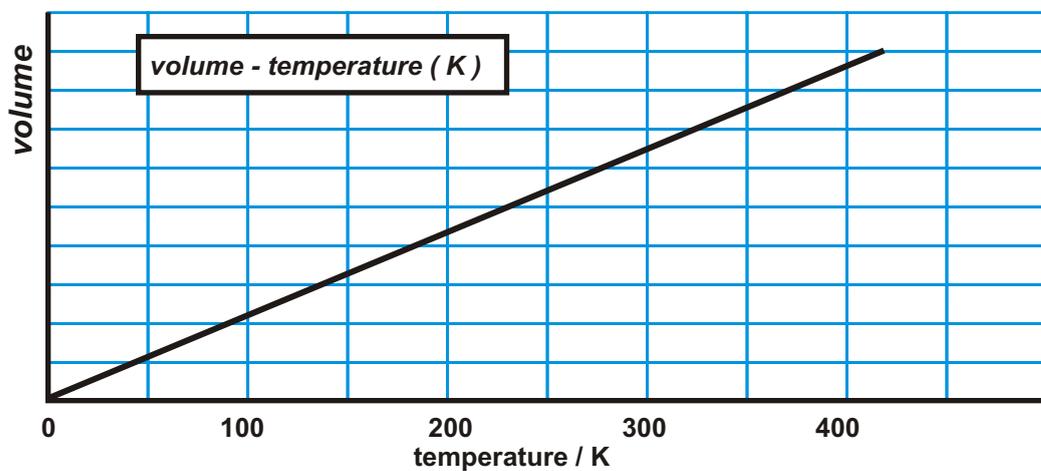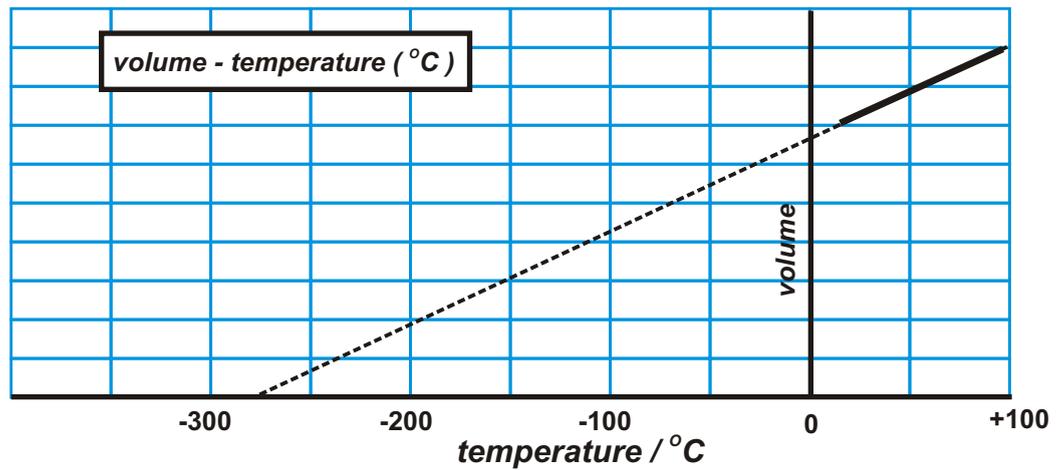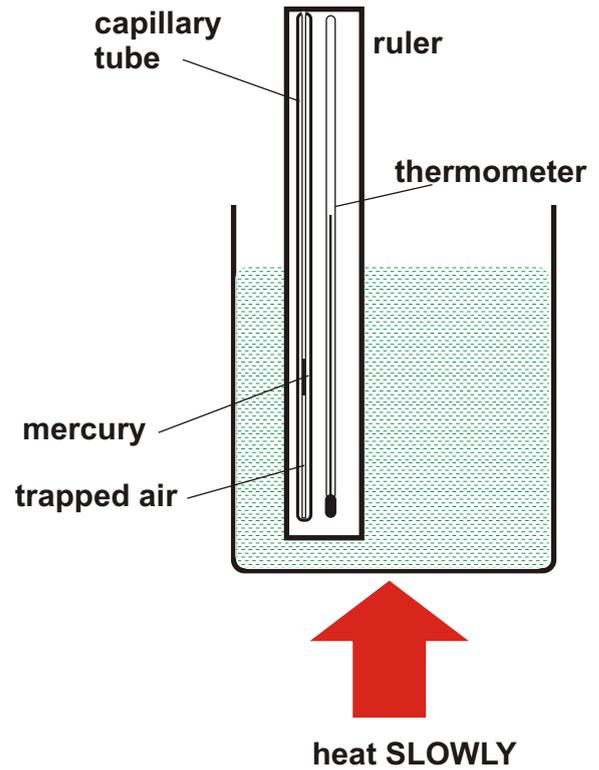
temperature / K

Pressure Law

The pressure of a fixed mass of gas at constant volume varies directly as the temperature of the gas measured on the Kelvin scale of temperature.

$$p \propto T_K$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

thermometer

bourdon pressure gauge

air

water

Heat SLOWLY

pressure - Temperature (^{o}C)

pressure / Pa

-300   -200   -100   0   +100

Temperature / ^{o}C

pressure - Temperature (K)

pressure / Pa

0   100   200   300   400   500

Temperature / K
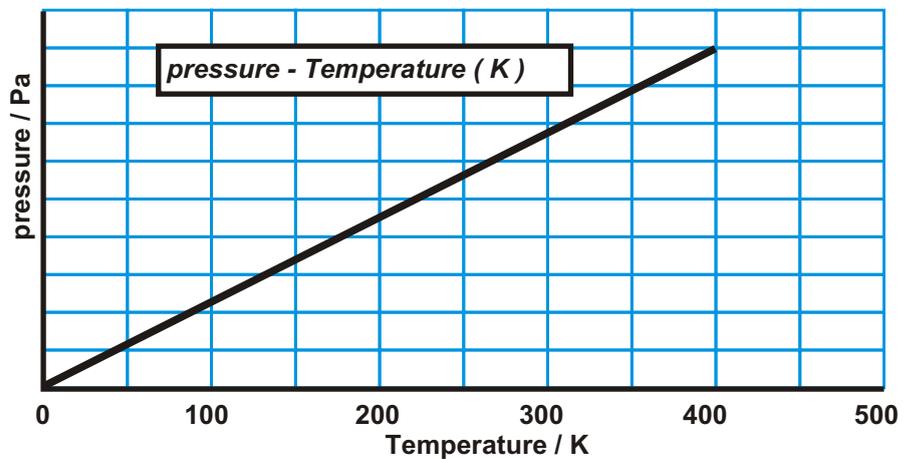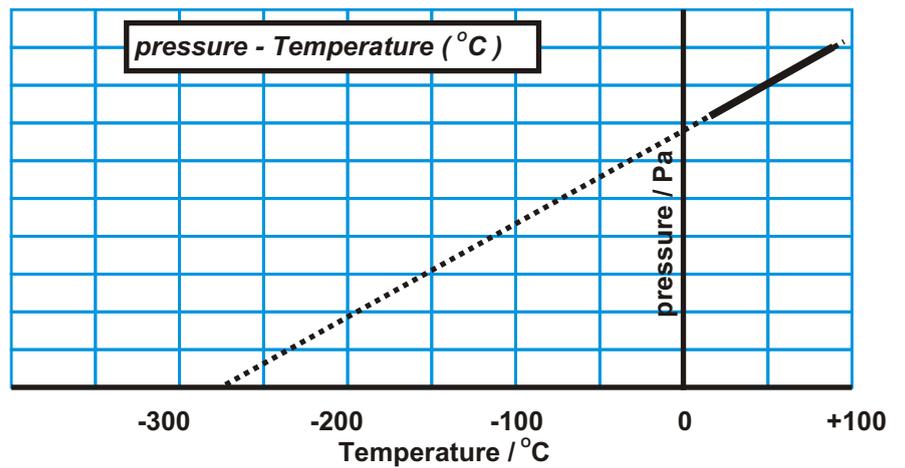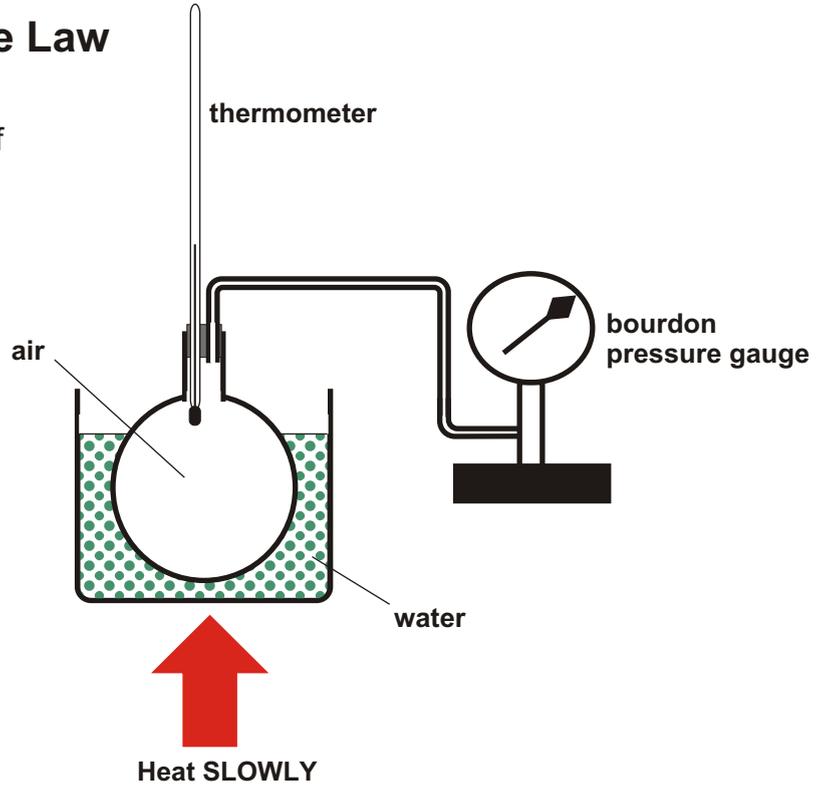
The General Gas Law

All the gas laws can be combined into one relationship.

$$\frac{p_1 \times V_1}{T_1} = \frac{p_2 \times V_2}{T_2}$$

Kinetic Model

Pressure The pressure exerted by a gas on the sides of its container is caused by gas particles colliding with the walls of the container. Each time the particle collides, the particle and the wall receive equal and opposite impulses. The particle bounces off the wall, the wall experiences a tiny force. Pressure is due to a large number of these collisions.
The magnitude of the pressure depends on the number of collisions per second and the average force per collision.

Volume The volume of a gas is the volume of its container and is the amount of space the particles have to move around in.

Bring walls closer together and particle takes less time to cover distance between bounces

gas particle bouncing between the walls of its container.
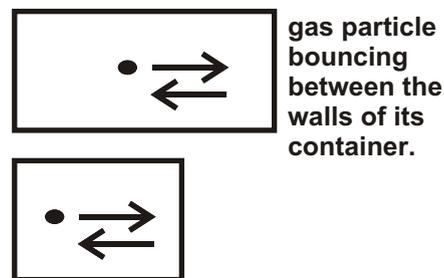
Temperature Adding heat energy to a gas increases the average kinetic energy and speed of the particles.

Gas Laws and the Kinetic Model.

Boyles' Law Increasing the volume available to a gas increases the time between collisions with the walls of its container. This decreases the number of collisions per second and with it, the pressure on the walls. The average force per collision depends on the average speed of the particles which remains the same at constant temperature.

Charles' Law Increasing the temperature causes the particles to speed up. This increases both the average force per collision and the number of collisions per second. To maintain a constant pressure, the volume increases to reduce the number of collisions per second. Volume varies directly as temperature for constant pressure.

Pressure Law Increasing the temperature causes the particles to speed up. This increases both the average force per collision and the number of collisions per second. If the volume remains constant, the pressure will increase. The pressure of a gas varies directly as the temperature for constant volume.

Example A syringe contains 50 cm^3 of air trapped air at a
temperature of 20°C. The syringe is immersed in boiling
water. Calculate the new volume of the trapped air.

Assuming the pressure remains constant.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{50}{293} = \frac{V_2}{373}$$

$$V_2 = \frac{50 \times 373}{293}$$

$$= \underline{63.7 \ cm^3}$$

V_1 = 50 cm^3

T_1 = 20°C

= 273 + 20

= 293 K

T_2 = 100°C

= 273 + 100

= 373 K

trapped air
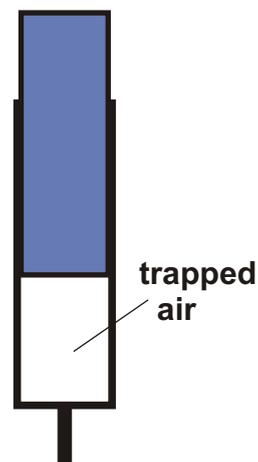
Example
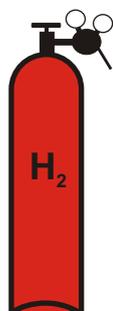
A gas bottle contains 0.2 m^3 Hydrogen gas at a
pressure of 200 x 10^5 Pa. It is used to deliver
Hydrogen to a chromatograph at the rate of
100 cm^3 per second at a pressure of 1 x 10^5 Pa.
How long should the bottle last before needing
to be changed?

H_2

Assume the temperature remains constant.

$$p_1 \times V_1 = p_2 \times V_2$$

$$200 \times 10^5 \times 0.2 = 1 \times 10^5 \times V_2$$

$$V_2 = 40 \ m^3$$

p_1 = 200 x 10^5 Pa

V_1 = 0.2 m^3

p_2 = 1 x 10^5 Pa

Gas produced from bottle = 40 - 0.2 = 39.8 m^3 as some gas is left in bottle

$$time = \frac{39.8 \times 10^6}{100} \ seconds$$

$$= 3.98 \times 10^5 s$$

$$= \underline{110.5 \ hours.}$$

Example An empty aerosol can contains 250 cm^3 propellant gas at a temperature of 20 $^{\circ}$C and a pressure of 1.5 x 10^5 Pa. For safety reasons, the can is designed to burst when the pressure inside the can is 3 x 10^5 Pa. Find the maximum temperature it can be safely exposed to.

Assuming the volume of the can remains constant

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$\frac{1.5 \times 10^5}{293} = \frac{3.0 \times 10^5}{T_2}$$

$$T_2 = \frac{293 \times 3.0 \times 10^5}{1.5 \times 10^5}$$

$$= 586 \ K$$

$p_1 = 1.5 \times 10^5 \ Pa$

$T_1 = 20^{\circ}C$

$\quad = 20 + 273 = 293 \ K$

$p_2 = 3.0 \times 10^5 \ Pa$

The aerosol can will remain intact up to a temperature of 586 K or 313 $^{\circ}$C

Higher Physics : Electricity and Electronics

Summary Notes

Electric fields and resistors in circuits

Electric Fields

An electric field is a volume of space where a charged object will experience a force. Electric fields are found around charged objects.

Electric fields can be 'mapped' using the concept of lines of force. The lines give the direction of the force on a positive charge. The magnitude of the force is worked out from how closely the lines are packed together.
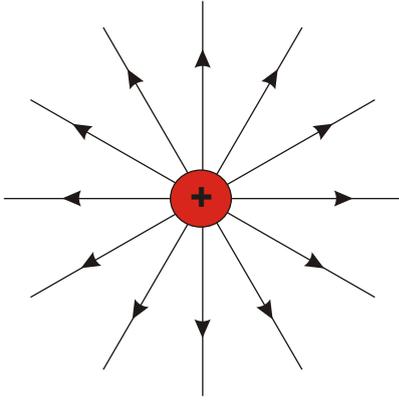
Weak positive charge
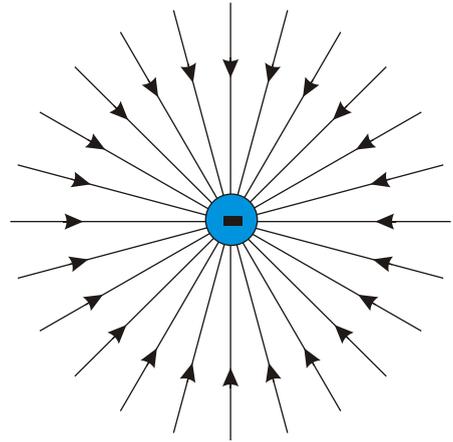
Strong negative charge
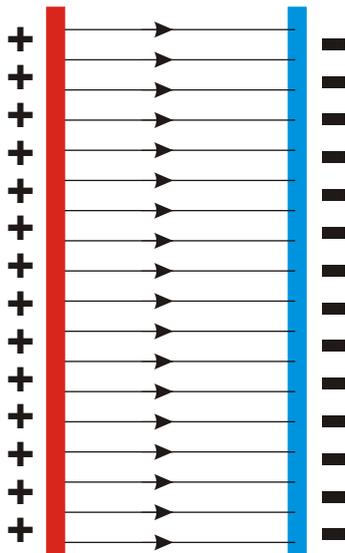
Uniform field between
two charged metal plates
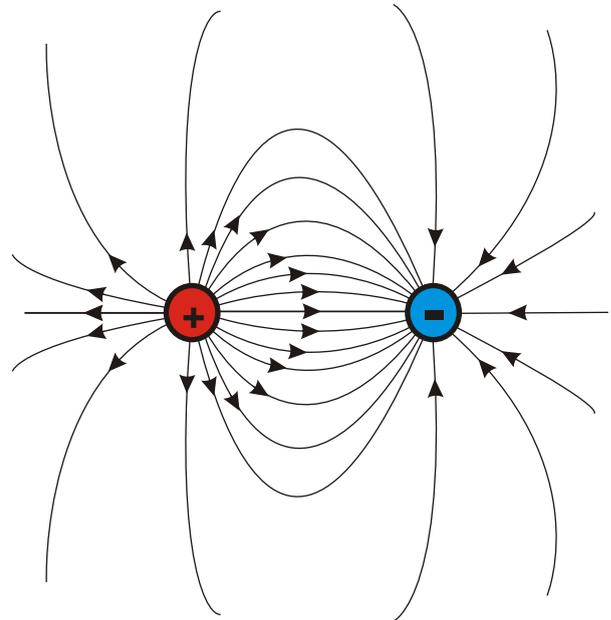
Dipole field

Potential Difference

it does not matter
what path is taken
between A and B

B

A
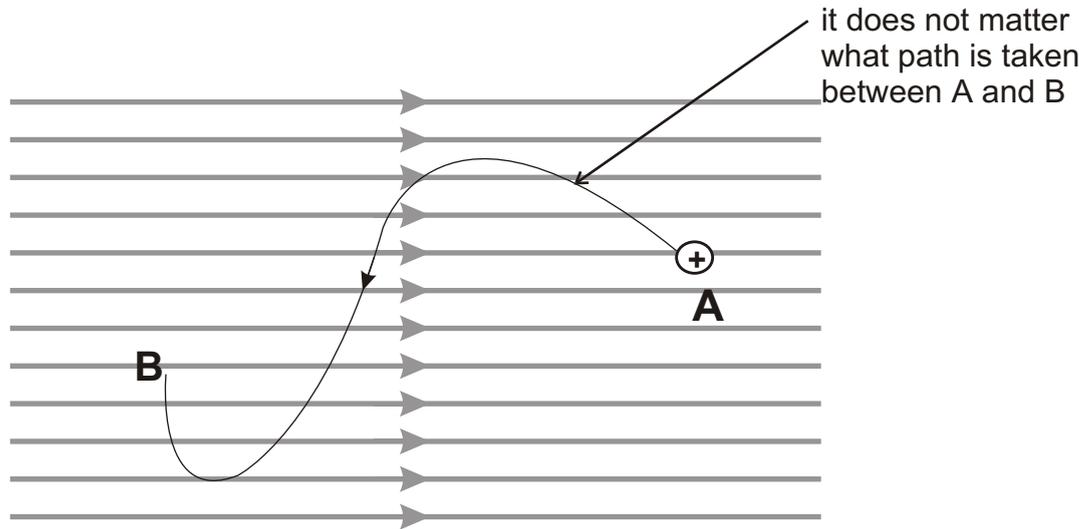
A charge in an electric field experiences a force. This means that moving a charge
in an electric field requires work to be done.
If 1 Joule of work is done moving 1 coulomb of positive charge between A and B,
then the potential difference between A and B is 1 Volt.
The work done in moving Q Coulombs of charge through a potential difference
of V volts is given by.

$$E_w = Q V$$

$$V = \frac{E_w}{Q}$$

The potential difference between two points is the work done per coulomb of charge
when charge is moved between those two points.

anode
+ V

cathode
0 V

deflection
plates

electron gun
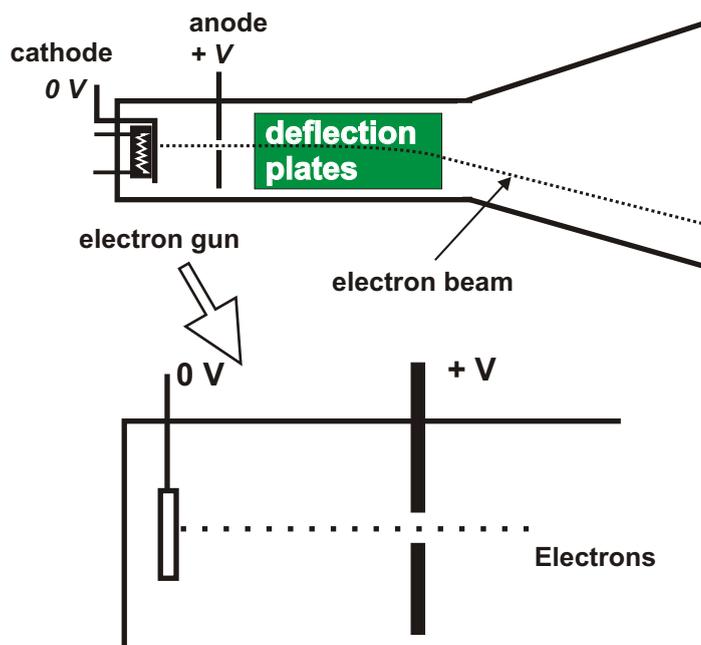
electron beam

A CRO tube uses electrostatic
deflection plates to move a beam
of electrons across a screen.
The electrons are provided by an
arrangement called an electron
gun.

0 V   + V

Electrons

Electrons are emitted from a heated
cathode and pulled across by a high
potential difference to the anode.
The electrons pass through a gap
in the anode to form a beam.
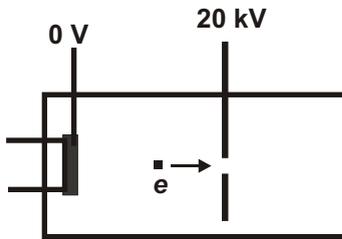
Example: The potential difference between the anode and the cathode of an electron gun is 20 kV. Calculate the speed of an electron when it reaches the anode.

Work done by the field moving the electron is converted to kinetic energy

$$Q\,V = \tfrac{1}{2}\,m\,v^2$$

$$1.6 \times 10^{-16} \times 20000 = 0.5 \times 9.11 \times 10^{-31} \times v^2$$

$$v = \left(\frac{1.6 \times 10^{-19} \times 20000}{0.5 \times 9.11 \times 10^{-31}} \right)^{\tfrac{1}{2}}$$

$$= 5.9 \times 10^{7} m\ s^{-1}$$

$Q = e = 1.6 \times 10^{-16}\ C$

$m = 9.11 \times 10^{31}\ kg$

$V = 20\ kV$

$= 20000\ V$
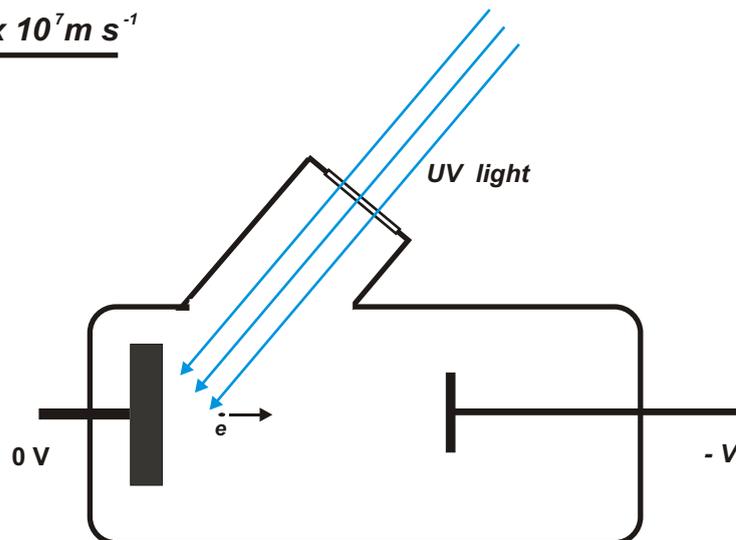
When certain metal surfaces are illuminated by UV light, electrons are emitted with an amount of kinetic energy. By subjecting the electrons to an opposing field it is possible to measure the maximum kinetic energy. This is equal to the minimum potential difference required to stop the electrons.

In one case the required pd was 6.3 V.

maximum kinetic energy = work done by field in stopping the electron

$$= Q\,V$$

$$= 1.6 \times 10^{-19} \times 6.3$$

$$= 1.08 \times 10^{-18}\ J$$

Electric Fields and conductors.

Conductors contain electric charges which are easily moved (*electrons*). When a conductor is exposed to an electric field, these charges will move.
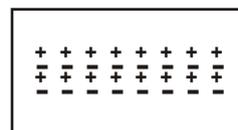
Conductor : positive and negative charges uniformly distributed
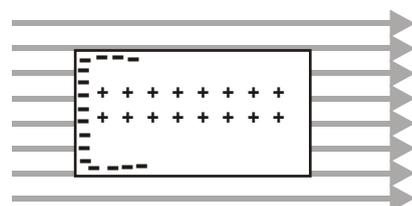
in an electric field, negative charges move in response to the force exerted by the field.

(battery, mains)
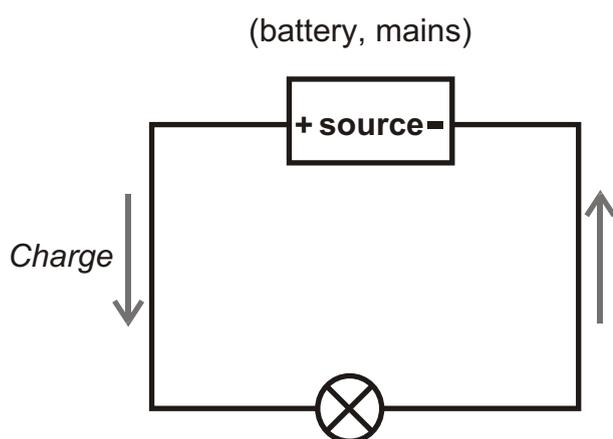
+ source −

Charge

Work is done by a source when electric charges are moved round a circuit. The work done comes from the electrical energy given to the charge as it passes through the source.

The electrical energy given to each coulomb of charge as it passes through the source is termed the e.m.f of the source (e.m.f - electromotive force)
E.m.f is measured in Volts ($J\ C^{-1}$)

The work done moving Q coulombs of charge round a circuit is
When dealing with circuits, we usually describe charge in terms of current

$$W = Q\ V \quad \text{where V is the pd across the source.}$$

$$Q = I\ t \quad \text{I is the current flowing in the circuit}$$

Substituting :

$$W = I\ t\ V$$

This is more familiar as

$$\frac{W}{t} = P = V\ I$$

$P = V\ I$ and $W = Q\ V$ *are equivalent expressions.*

Internal Resistance
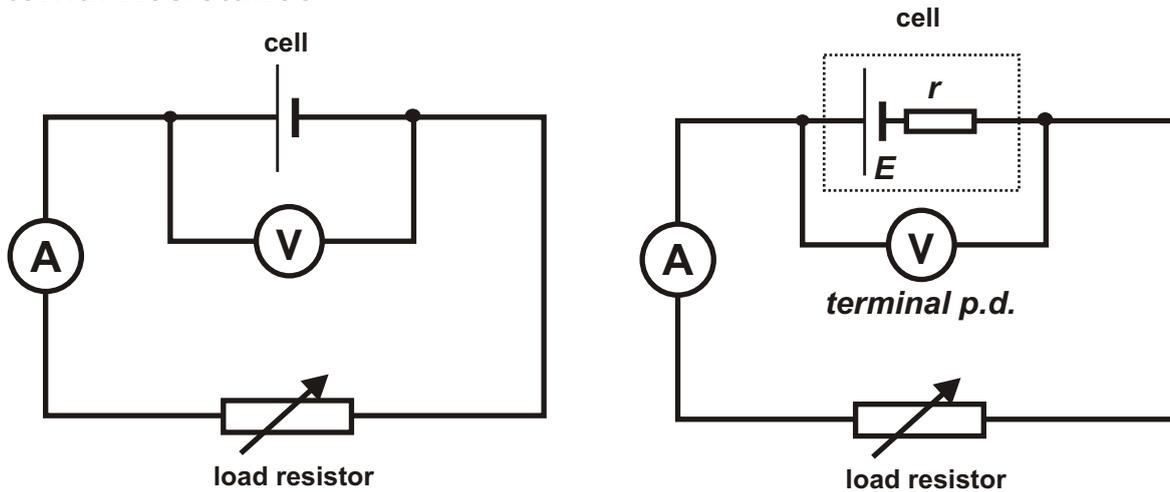
The potential difference across the terminals of a battery, or any other source, decreases as the current drawn from the source increases.
The behaviour of the source can be predicted if we assume the source consists of a source of constant e.m.f with a small internal resistor in series with it.

V *p.d. across terminals of source (terminal p.d.)*

E *e.m.f of source*

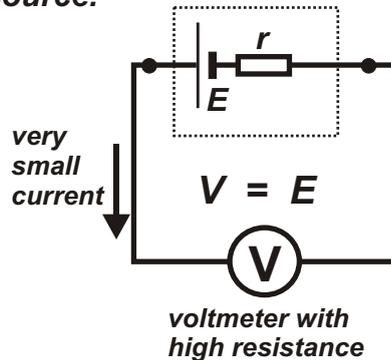**r** *internal resistance of source*

I *current drawn from source*
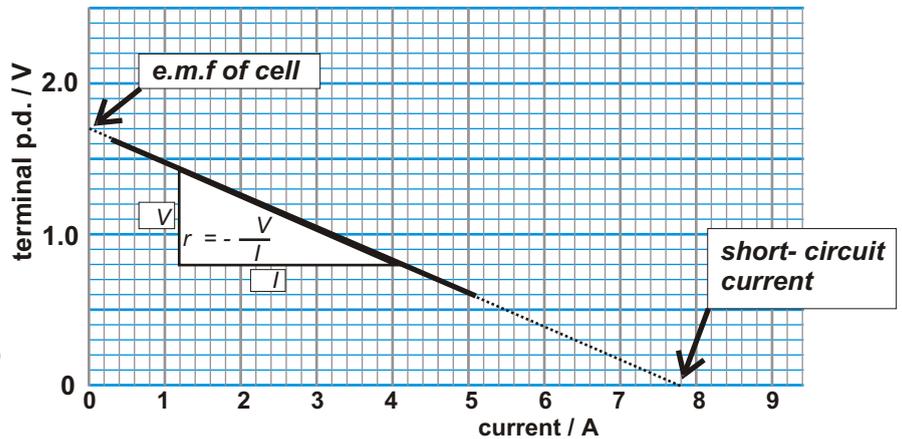
$$V = E - Ir$$

Ir is often termed the 'lost volts'

The maximum current drawn from a source occurs when the terminal p.d. falls to zero. This is termed the short - circuit current and is given by

$$I_s = \frac{E}{r}$$

V = E when no current is being drawn from source.

The e.m.f of a source can be found by measuring the p.d. across its terminals using a voltmeter with high resistance as very little current is drawn from the source.

Example When a voltmeter is connected across the terminals of a battery it reads 9.2 V. when the battery is connected in series with a 5 ohm resistor, the voltmeter reads 8.5 V. Find   (a) the e.m.f of the battery.

        (b) the internal resistance of the battery;

        (c) the maximum current which can be drawn from the battery.

(a) The e.m.f of the battery is 9.2 V The voltmeter draws virtually no current so the reading on the voltmeter is equal to the e.m.f of the battery.

(b)

$$V = E - Ir$$
$$8.5 = 9.2 - 1.7\,r$$
$$r = \frac{9.2 - 8.5}{1.7}$$
$$= 0.41\ ohms$$

$V = 8.5\ V$

$E = 9.2\ V$

$I = \dfrac{V}{R} = \dfrac{8.5}{5}$

$\quad = 1.7\ A$

(c)   The maximum current which can be drawn from the battery is the short circuit current when the terminal p.d. falls to zero volts.

$$0 = E - Ir$$
$$I_s = \frac{9.2}{0.41}$$
$$= 22.4\ A$$

Internal resistance and Power transfer to external resistive loads
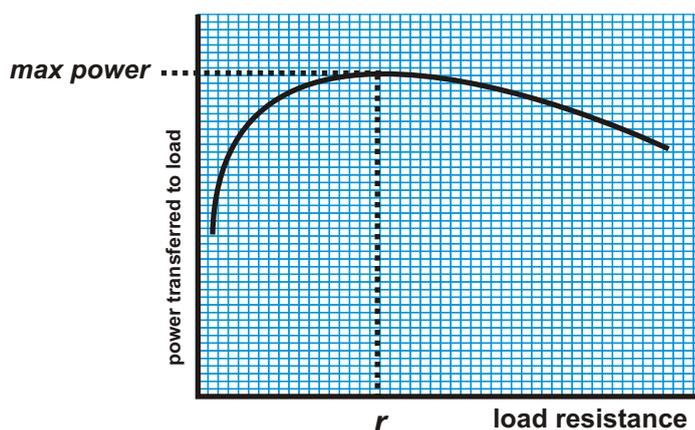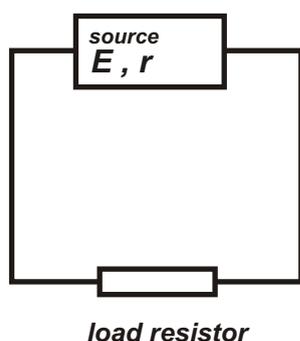
The maximum power transferred between a source and an external circuit occurs when the resistance of the external circuit is equal to the internal resistance of the source. The p.d. across the circuit when this happens is ½ E..
The maximum voltage transfer occurs when the external resistance is much higher than the internal resistance of the source. .

Resistors in Series and Parallel

Conservation of energy : The energy suppllied to unit charge as it moves round the circuit is equal to the work done moving the charge round the circuit.
The energy supplied is equal to the sum of the source e.m.f.s round the circuit. The work done is equal to the sum of the potential differencesround the circuit.
The sum of the e.m.f.s = sum of the p.d.s round the circuit.
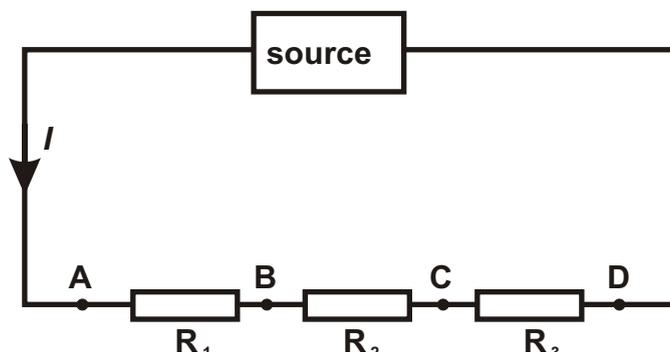
In the circuit above

$$V_{AD} = V_{AB} + V_{BC} + V_{CD} \qquad \textit{Conservation of energy}$$

Dividing both sides by I, the current flowing round the circuit.

$$\frac{V_{AD}}{I} = \frac{V_{AB}}{I} + \frac{V_{BC}}{I} + \frac{V_{CD}}{I}$$

$$\underline{R_T = R_1 + R_2 + R_3}$$

The combined resistance of a number of resistors in series is equal to the sum of the individual resistances.

Conservation of Charge

The current flowing into a junction must equal the current flowing out of a junction.

$$I_T = I_1 + I_2 + I_3$$
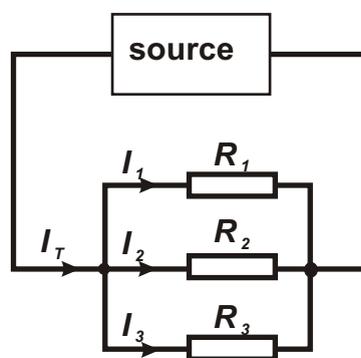
The p.d. across all the resistors is **V**

*dividing both sides by **V***

$$\frac{I_T}{V} = \frac{I_1}{V} + \frac{I_2}{V} + \frac{I_3}{V}$$

$$\underline{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad \textit{Where } \; R_T = \frac{V}{I_T}$$

The combined resistance of parallel resistors is given by

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Example.
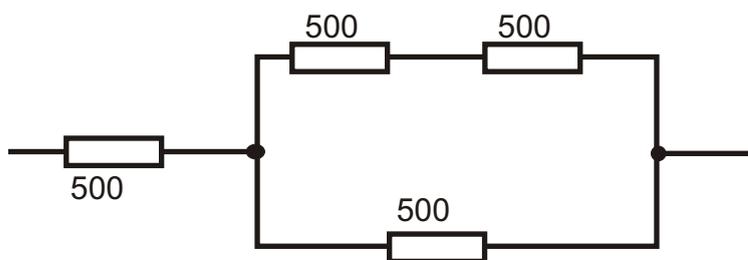
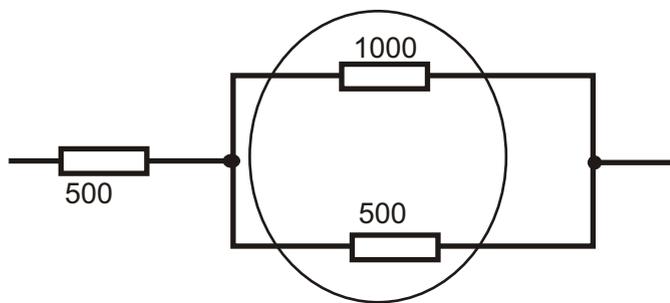Find the combined resistance of the arrangement shown above.

Stage 1

500 + 500 = 1000
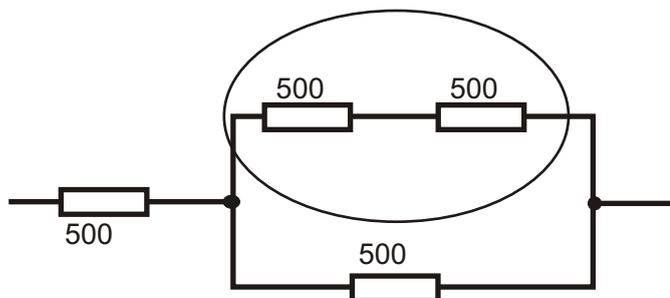
Stage 2

$$\frac{1}{R_P} = \frac{1}{500} + \frac{1}{1000}$$
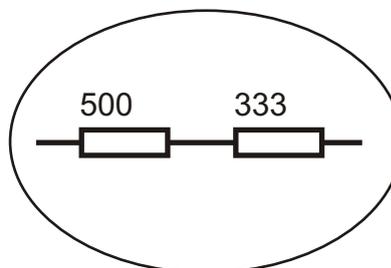
$$R_P = 333$$
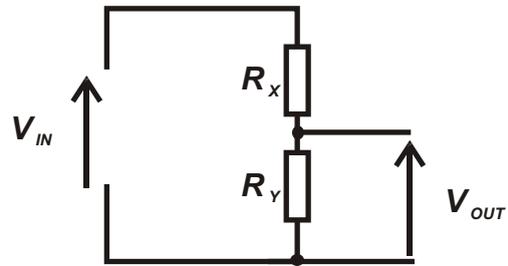

Stage 3.

$$R_T = 500 + 333$$

$$= 833$$

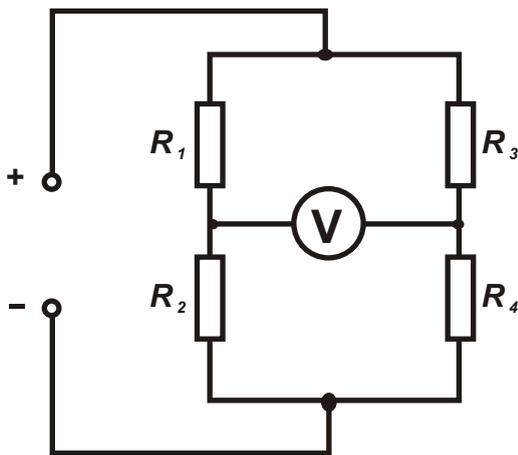*Total resistance of above arrangement = 833 ohms.*

This type of problem is solved in stages, tackling the parallel arrangements first and progressively simplifying.

Wheatstone Bridge

A Wheatstone Bridge consists of two potential dividers in parallel. The outputs of the two potential dividers is bridged by a voltmeter.

Potential Divider

Balanced Wheatstone Bridge

The Wheatstone Bridge is balanced when the voltmeter reads zero.
When balanced, the ratio of the resistors in each potential divider is the same.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

The Wheatstone Bridge was originally used as a means of measuring resistance in a situation where accurate resitors were available but no accurate meters. The bridge only requires a meter capable of detecting a current. No current measurements are needed.
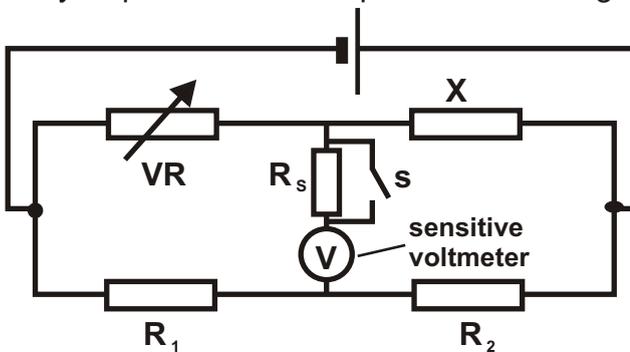
Typical set up : X is the unknown resistor. With switch S open, the voltmeter is less sensitive. R_s protects the voltmeter from large currents in the early stage of adjustment. VR is adjusted till voltmeter reads zero. Switch S is closed, making the voltmeter very sensitive. Final adjustments are made to VR. Resistance is found from :
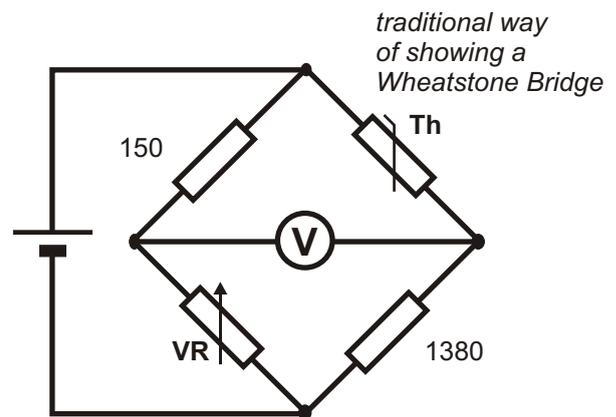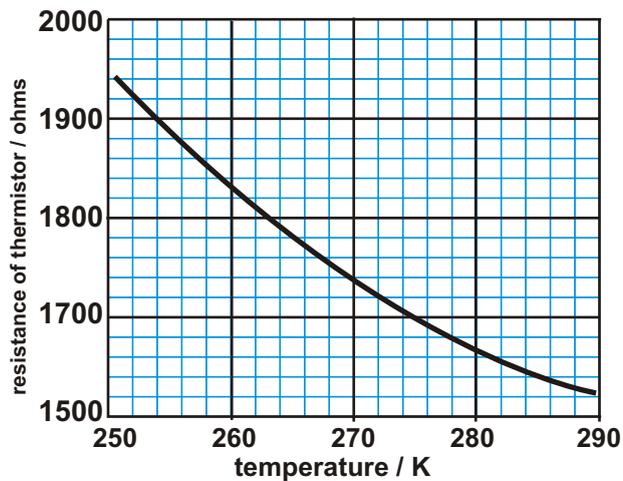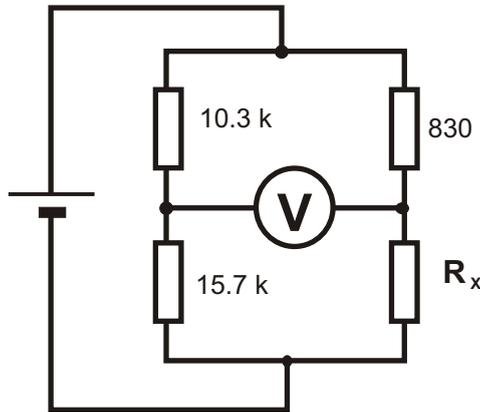
$$X = \frac{(VR) \times R_2}{R_1}$$

The balanced condition is unaffected by the voltage of the supply.

Example

Find the value of R_x to balance the bridge.

$$\frac{R_x}{830} = \frac{15.7}{10.3}$$

$$R_x = \frac{830 \times 15.7}{10.3}$$

$$= \underline{1265}$$

10.3 k   830

V

15.7 k R_x

traditional way of showing a Wheatstone Bridge

150   Th

V

VR   1380

A thermistor, Th, connected into a Wheatstone Bridge circuit as shown, is immersed in melting ice. The variable resistor VR is adjusted to balance the bridge. The graph shows the variation in the resistance of the thermistor over a small temperature range. Calculate the resistance of VR.

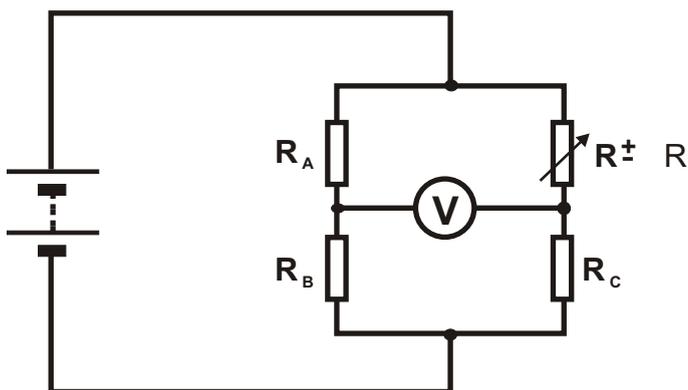*From the graph the resistance of the thermistor @ 273 K (0°C) is 1707 .*

*Condition for Balance*

$$\frac{VR}{150} = \frac{1380}{1707}$$

$$VR = \frac{150 \times 1380}{1707}$$

$$= \underline{121}$$

The Unbalanced Wheatstone Bridge.
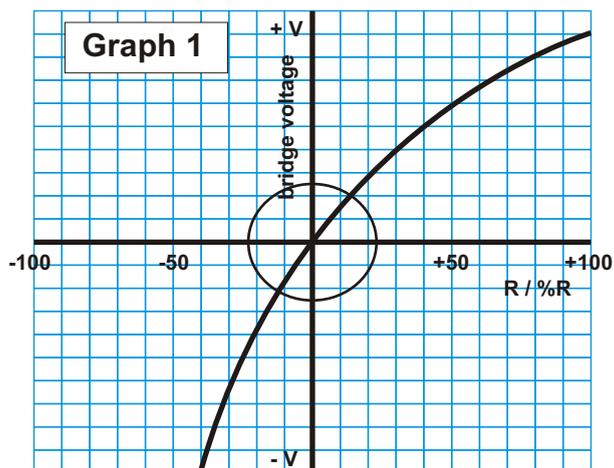
Graph 1

If a Wheatstone Bridge is balanced then the value of one of the resistors is changed, A voltage will be recorded on the voltmeter. For large variations, the graph is a curve as shown in Graph 1. For small variations: no more than 5% change in the value of the resistor, the graph is a straight line through the origin (Graph 2).
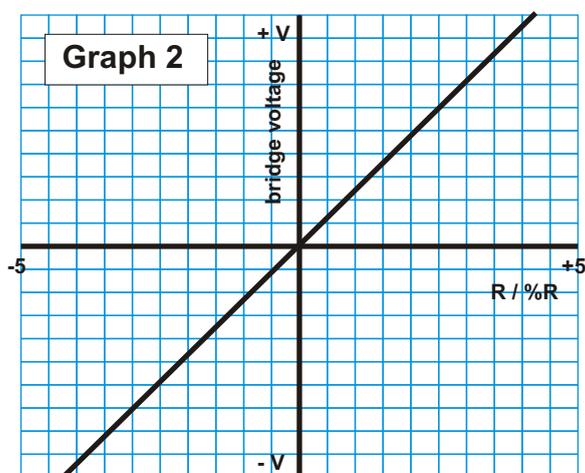
For small changes in the value of one of the resistors in a balanced Wheatstone Bridge, the unbalanced voltage across the bridge varies directly as the change in resistance.

Graph 2

Small area round origin

$$V = constant \times R$$

Th e Wheatstone Bridge circuit is used extensively in measurement circuits incorporating resistive sensors. These include strain gauges and resistance thermometers.

–plastic backing
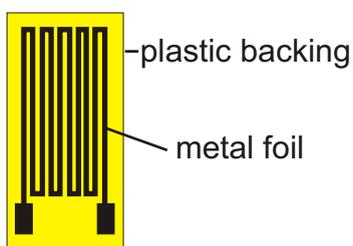
metal foil

strain gauge consists of a length of metal foil bonded onto a flexible plastic backing sheet.
The resistance of the foil (usually 120 ohms) depends on the length of the foil. The change of resistance depends on the stretch in the gauge.
When stuck on metal, any stretch in the metal caused by forces changes the resistance of the strain gauge,

Alternating current and voltage

Measuring frequency using a CRO.

The speed of the electron beam as it moves horizontally across the face of the screen is controlled by the **time base** control. This is usually scaled in seconds per screen division. Once the signal is stationary on screen, The number of screen divisions per cycle of signal is measured. Multiplied by the time base setting, this measures the period of the signal. Frequency is calculated from 1 / period.

time base - s/div.
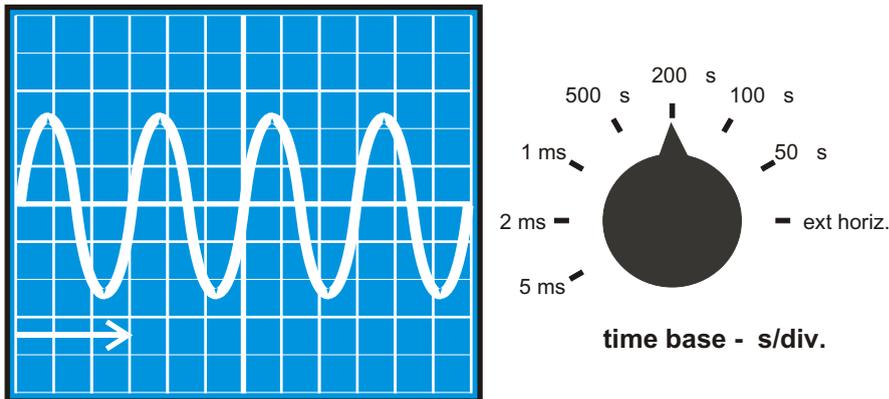
$$\text{frequency} = \frac{1}{\text{period}}$$

$$= \frac{1}{3 \times 200 \times 10^{-6}}$$

$$= \underline{1667 \text{ Hz}}$$

period = time for 1 cycle
= time to move 3 divisions
= 3 x 200 x 10⁻⁶ s

AC Supplies
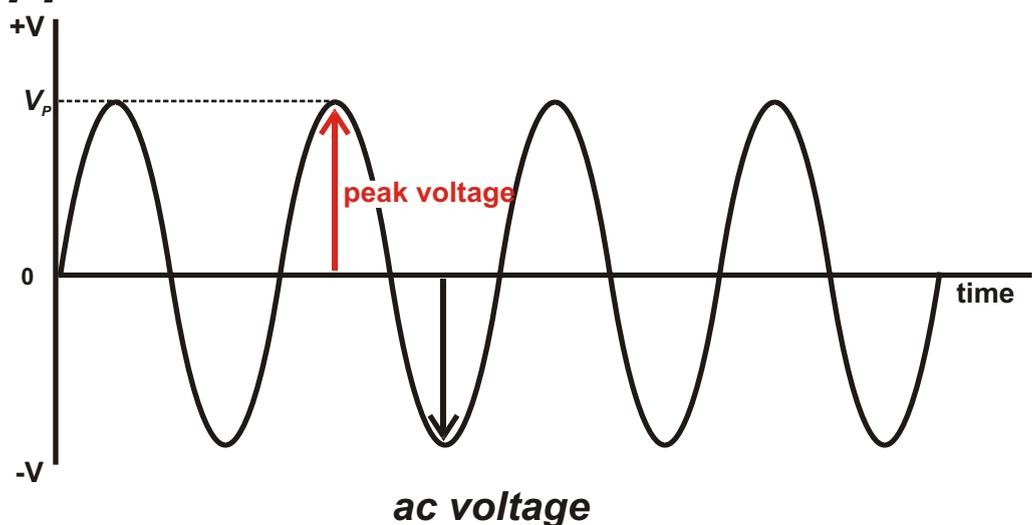
ac voltage

The voltage of an a.c. supply changes between positive and negative. Over time, the voltage spends the same time as a positive voltage as it does as a negative voltage. The average voltage, over time, is zero.
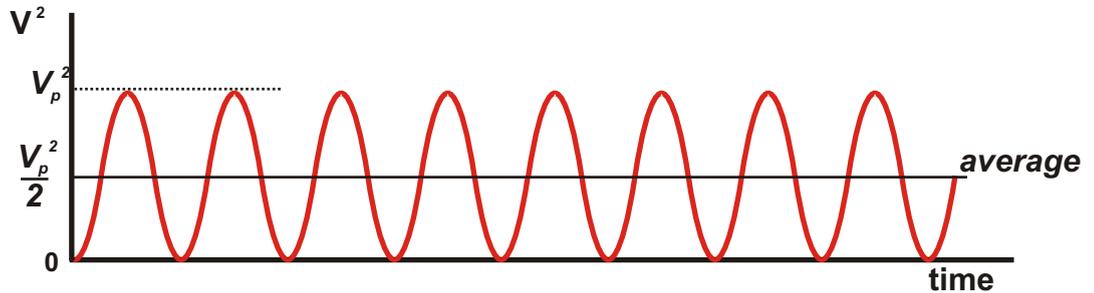
Power calculations and ac.

For dc supplies $P = \dfrac{V^2}{R}$

For ac supplies $P = \dfrac{average\ V^2}{R}$

We can find a average value for V^2.
For sinusoidal a.c. voltage

$V = V_P \sin \omega t$

$V^2 = V_p^2 \sin^2 \omega t$ The graph of this function is shown above.

The average value over time is $\dfrac{V_p^2}{2}$

This means that for ac $P = \dfrac{V_p^2}{2R} = \dfrac{V_{rms}^2}{R}$

Where : $\boxed{V_{rms} = \dfrac{V_P}{\sqrt{2}}}$ V_P *is the peak ac voltage*

The r.m.s voltage of an a.c. source is the equivalent d.c. voltage which will produce the same heating effect when applied across a resistive load.

Example: Mains electricity is supplied at 230 V r.m.s. at a frequency of 50 Hz. Calculate the peak voltage.

$V_{rms} = \dfrac{V_{peak}}{\sqrt{2}}$

$V_{peak} = \sqrt{2} \times V_{rms}$

$\qquad = 1.41 \times 230$

$\qquad = \underline{325\ V}$

Measurement of a.c. voltage and current

A.c. voltmeters and ammeters are calibrated to read *r.m.s.* voltage and current. All a.c. supply voltages are quoted in r.m.s. values.

R.m.s. voltages and current are unaffected by the frequency of the a.c. supply. Some common circuit components like capacitors or inductors are affected by the frequency of the supplied a.c.

Resistors and frequency.

The resistance of a resistor is *unaffected* by the frequency of an a.c. supply. This can be demonstrated using the circuit shown below. Both meters read r.m.s values.

$$R = \frac{V}{I} = \frac{V_{rms}}{I_{rms}}$$
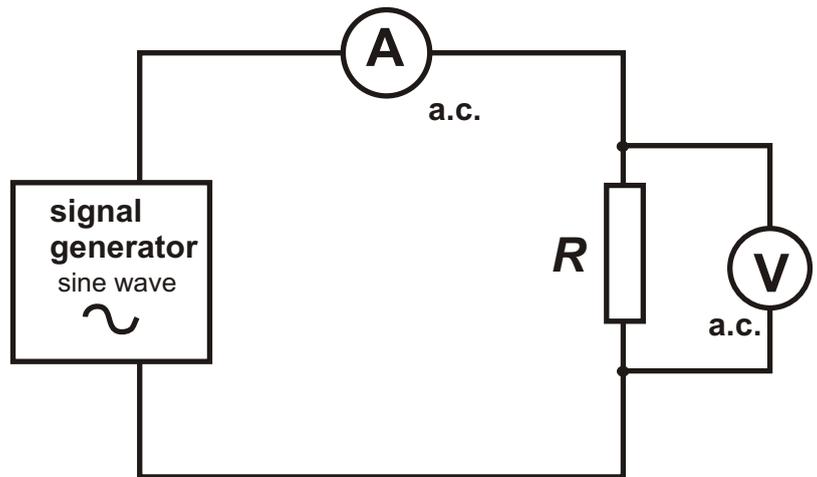
Most circuit components, used in electronics, are designed to operate within a range in voltage and can be fatally damaged if the range is exceeded. When dealing with a.c. voltage, we need to be aware of the peak value of the a.c. rather than the quoted os voltmeter measured r.m.s value.

Diodes. Diodes are used to convert a.c. current into d.c. current : rectification. The diode 'cuts off' the negative part of the a.c. cycle.

Semiconductor diodes can only be reversed biassed up to their peak inverse voltage (PIV) rating. When this is exceeded, the diode is fatally damaged.

Forward biassed
diode conducts

Reverse biassed
diode does not conduct

rectified a.c. voltage
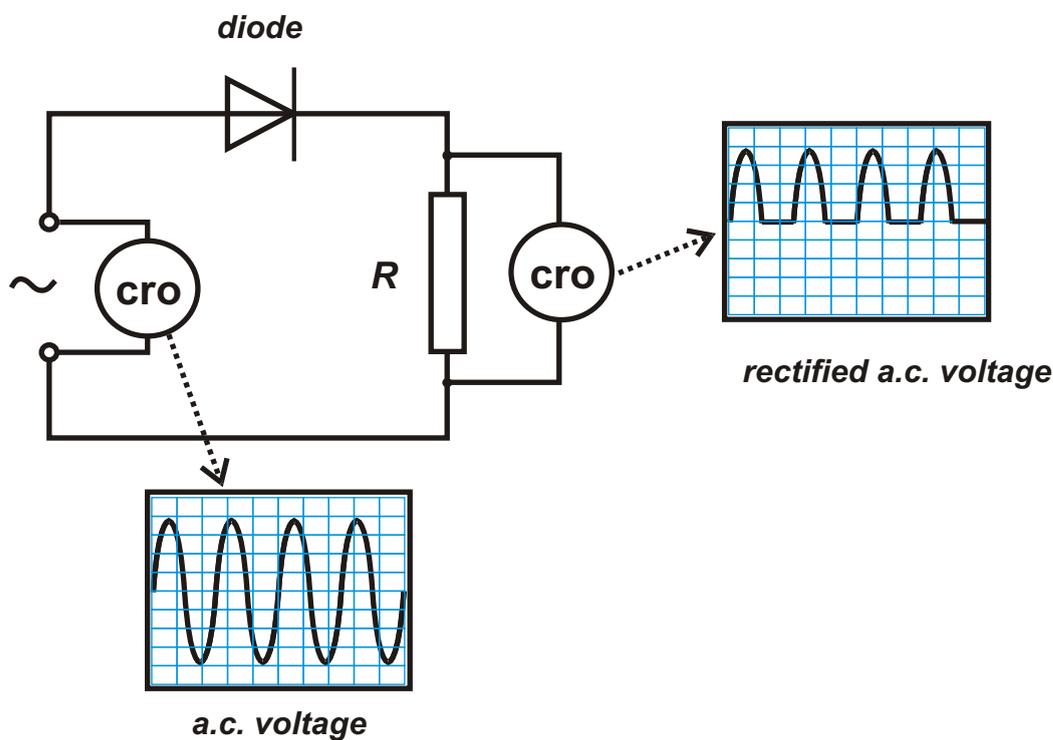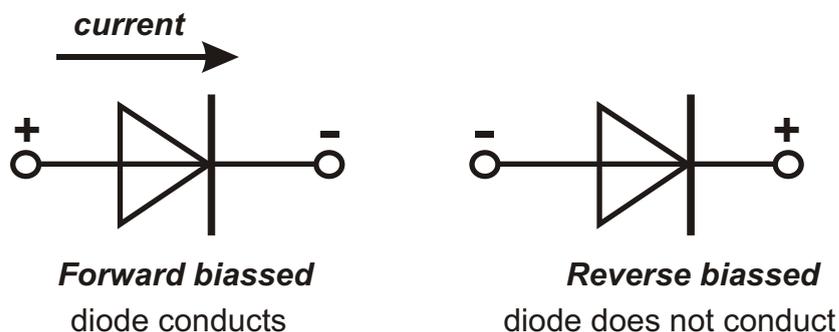
a.c. voltage

Example Calculate the maximum a.c. supply voltage which can be rectified by a diode with a PIV rating of 100 V.

In this situation, V_P must not exceed 100 V

$$V_{rms} = \frac{V_P}{\sqrt{2}}$$
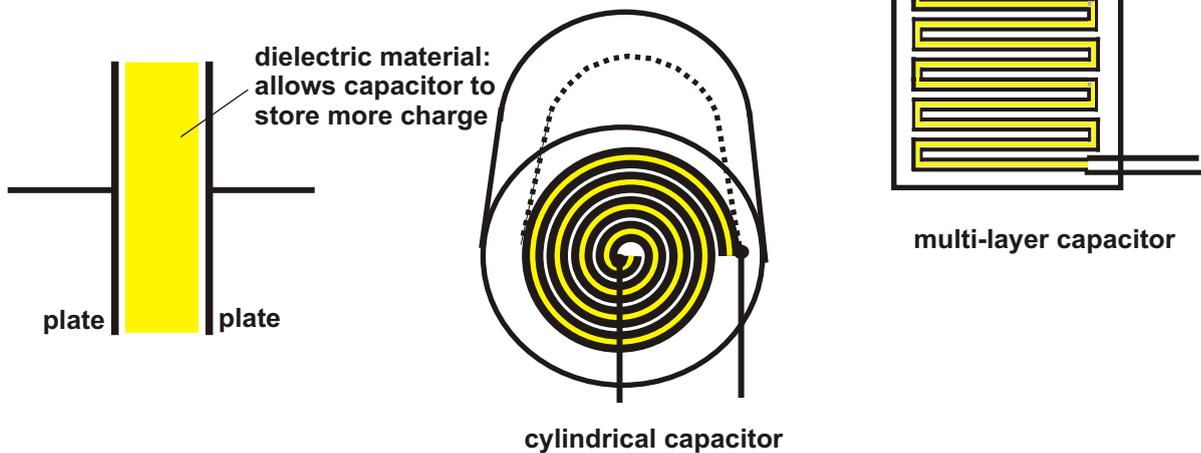$$= \frac{100}{1.41}$$
$$= 70.7 \text{ V}$$

Maximum supply voltage is 70.7 V r.m.s.

Capacitance

Introduction

A capacitor is a device which stores electric charge. Most capacitors consist of two sheets of metal foil separated by a thin layer of dielectric material.
The arrangement can be rolled into a
cylinder or folded into layers. In this way,
large areas of foil can be incorporated into a
small space.

dielectric material:
allows capacitor to
store more charge

plate   **plate**

cylindrical capacitor

multi-layer capacitor

Charge is stored in a capacitor by moving charge from one plate to the other. The work done required to move the charge(electrons) is provided by an external source.
As more charge is moved, the work done to move extra charge has to increase to overcome the repulsion of the charges already stored on the plates. This causes the p.d. across the plates to increase as the stored charge increases (p.d. is the work done moving unit charge).
The charging stops when the p.d. across the capacitor is equal to the p.d. across the source: there is no more energy available in the source to move charge.

V

e^- e^-
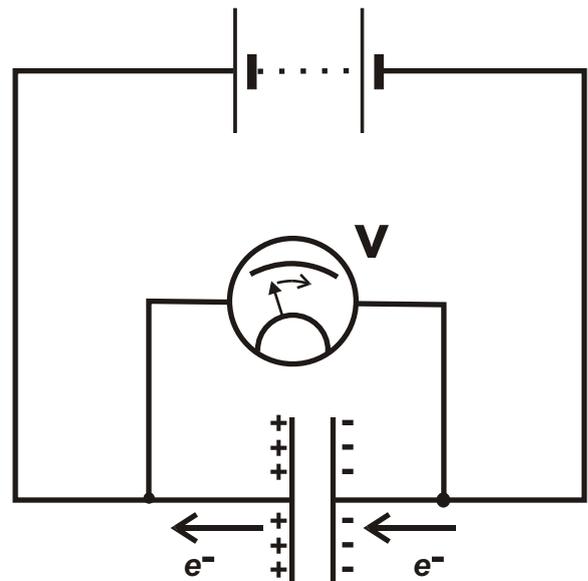
Charge cannot pass through the capacitor, it can only be moved externally though a circuit connecting the plates. Once charged, no more current will flow in the circuit.
A charged capacitor acts like a battery, and can be discharged through an external circuit. Charge does not pass through the capacitor so, unlike a normal source, the capacitor has no internal resistance. This means that it is possible to discharge enormous currents over tiny time intervals. The energy involved is small but due to the small time intervals, it is possible to create large power discharges.

Capacitance

Charge and p.d. across a capacitor

Switch, S, is set to *x*, and the capacitor charged up. The p.d. across the capacitor is noted. The switch is set to *y*, and the capacitor discharged through the coulombmeter. The charge collected by the coulombmeter is noted. This is repeated for different p.d. s across the capacitor
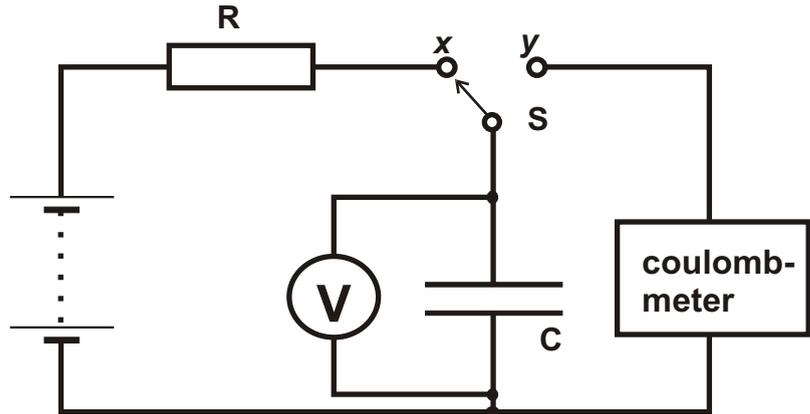
The quantity of charge stored in a capacitor varies directly as the p.d. across the capacitor.

$$Q = CV$$

C is the **capacitance** of the capacitor. Capacitance is measured in Farads (*F*).
One Farad is equal to one coulomb per Volt.
The Farad is a large unit, normal capacitors usually have values in microfarads (F) or pecofarads (*pF*) or nanofarads(*nF*).

$Q = It$ — charge stored in capacitor (y-axis)
p.d. across capacitor — V_c (x-axis)
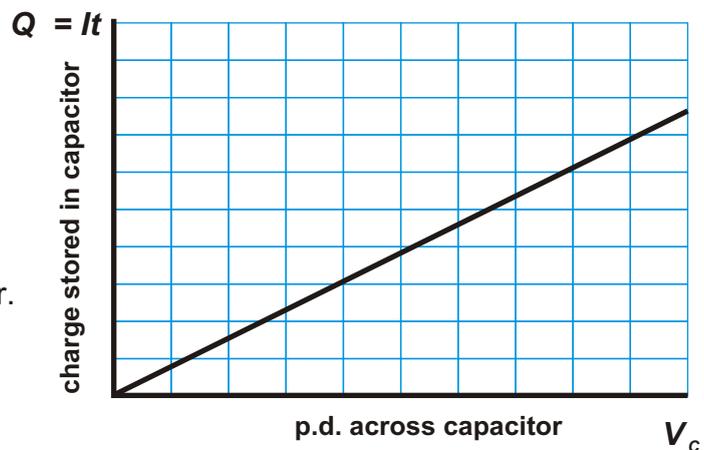
Example How much charge is stored in a 10000 F capacitor when the p.d. across it is 10 V?

$$Q = CV$$

$$= 10000 \times 10^{-6} \times 10$$

$$= 0.1\ C$$

$$1\ F = 1 \times 10^{-6}\ F$$

Energy stored in a capacitor

Work must be done to charge a capacitor. Once the plates of a capacitor have gained some charge, the stored charge repels more charge coming onto the plate. Work has to be done by the external source to overcome the repulsion and move charge onto the plates. The energy stored in the capacitor is equal to the work done charging the capacitor.
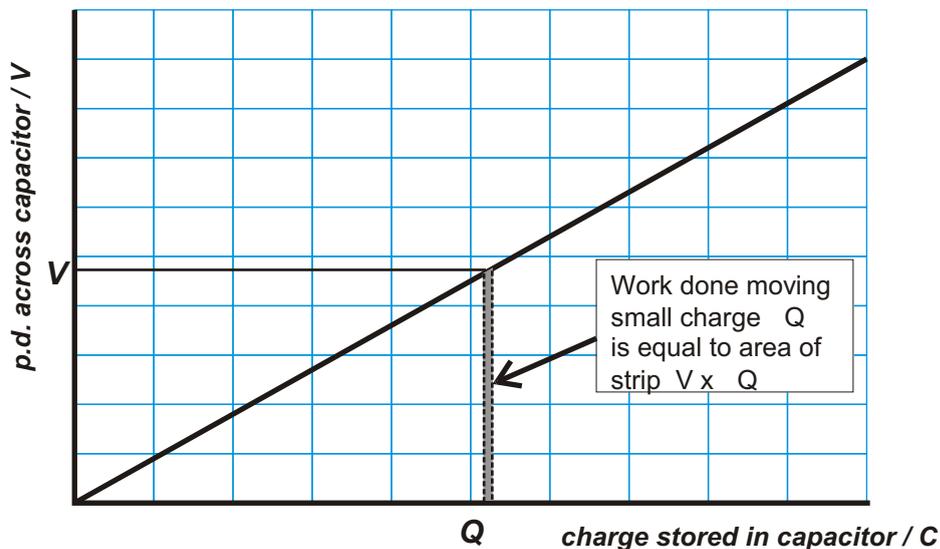
Suppose a capacitor is given a charge Q coulombs and that it has now got a p.d of V Volts across it.. A tiny amount of charge ΔQ is now moved from one plate to the other.
The work done in moving this tiny amount of charge is $V \times \Delta Q$.
$V \times \Delta Q$ Is the area of the small strip on the graph. If we charge up the capacitor by ΔQ coulombs each time, then the total work done will be the sum of the areas of the strips. This is simply the total area under the graph,

$$\text{work done charging a capacitor} = \tfrac{1}{2} Q \times V$$

$$\textbf{Energy stored in a capacitor} = \tfrac{1}{2} Q \times V$$

$$\text{substituting } Q = C \times V \qquad = \tfrac{1}{2} C \times V^2$$

$$\text{substituting } V = \frac{Q}{C} \qquad = \frac{Q^2}{2C}$$

The energy is stored in the stretched molecules of the material between the plates of the capacitor. Positive and negative charges in the material are pulled apart by the electric field between the plates.

Example How much energy is stored in a 1000 μF capacitor when there is a p.d of 6 V across it?

$$E = \tfrac{1}{2} C \times V^2$$

$$= 0.5 \times 1000 \times 10^{-6} \times 6 \times 6$$

$$= \underline{1.8 \times 10^{-2} \text{ Joules}}$$

Example A camera flash unit consists of a discharge tube powered by a 10 μF capacitor. The capacitor is charged up so that there is a p.d. of 300 V across it, and then discharged though the tube to produce a bright flash of light.

(a) How much energy is stored in the capacitor when it is fully charged?

(b) Calculate the average power output if the capacitor is fully discharged in 2 ms.

(a)

$$E = \frac{1}{2} C \times V^2$$

$$= 0.5 \times 10 \times 10^{-6} \times 300 \times 300$$

$$= \underline{0.\ 45\ Joules}$$

(b)

$$P = \frac{E}{t}$$

$$= \frac{0.45}{2 \times 10^{-3}}$$

$$= 225\ Watts$$

Average power output of the capacitor is 225 Watts.

This level of output will generate a powerful but short-lived flash of light, ideal for photography.

Example A capacitor is charged up using a constant current of 2 mA. After 12 seconds the p.d.. across the capacitor is 5 V. If the capacitor was fully discharged at the start of the charging, calculate the capacitance of the capacitor.

$$Q = CV$$

$$0.024 = C \times 5$$

$$C = \frac{0.024}{5}$$

$$= 0.0048 \ F$$

$$= \underline{4800} \ F$$

$$Q = I\,t$$

$$= 2 \times 10^{-3} \times 12$$

$$= 0.024 \ C$$
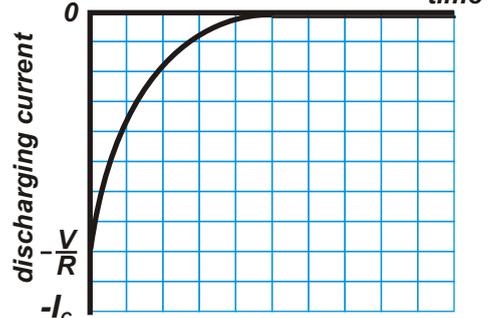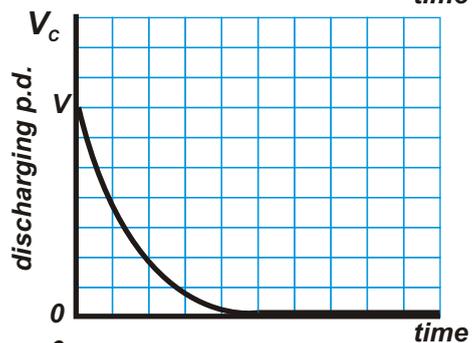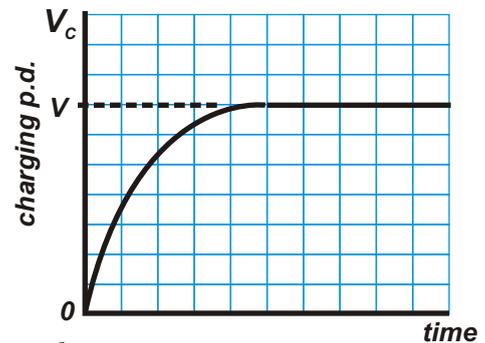
Charging / discharging a capacitor

Charging

+V

Capacitor is fully discharged at start

Battery has negligible internal resistance
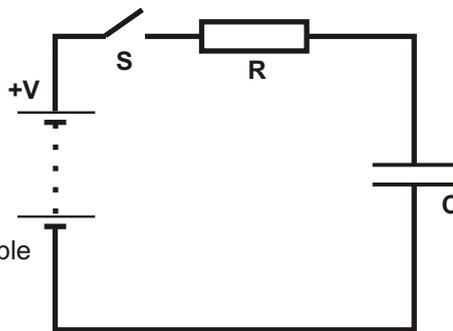
When switch S is closed, the p.d. across the capacitor is zero as it has no charge. The p.d. across the resistor is V. The current flowing through the resistor is V/R
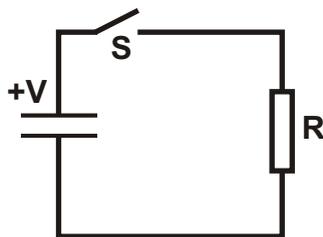*After a time, the capacitor has accumulated charge and the p.d. across it has increased to V_c. The current through the resistor has fallen to $(V - V_c)/R$.*
*When fully charged, the p.d. across the capacitor is a steady V. The current through the resistor is zero.*
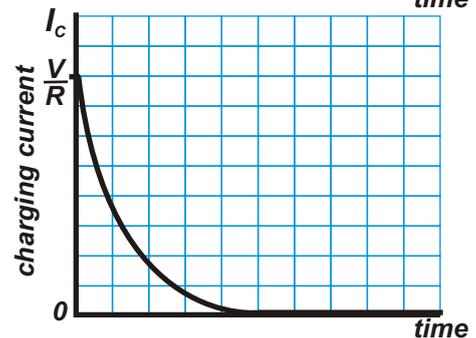
Discharging

The capacitor is fully charged with a p.d. of V across it.

When switch, S, is closed, the capacitor behaves like a small battery and discharges through the resistor. The p.d. falls till it is fully discharged. The discharge current is in the opposite direction to the charging current (hence -ve.). The current through the resistor starts at -V/R and falls in magnitude to zero when the capacitor is fully discharged.
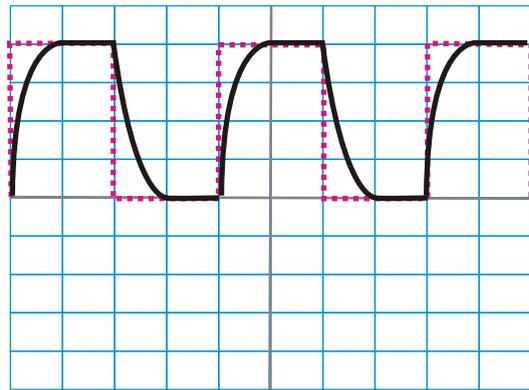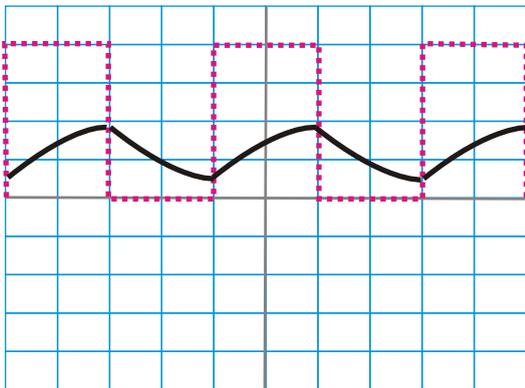
Effect of RC cicuits on square wave signals
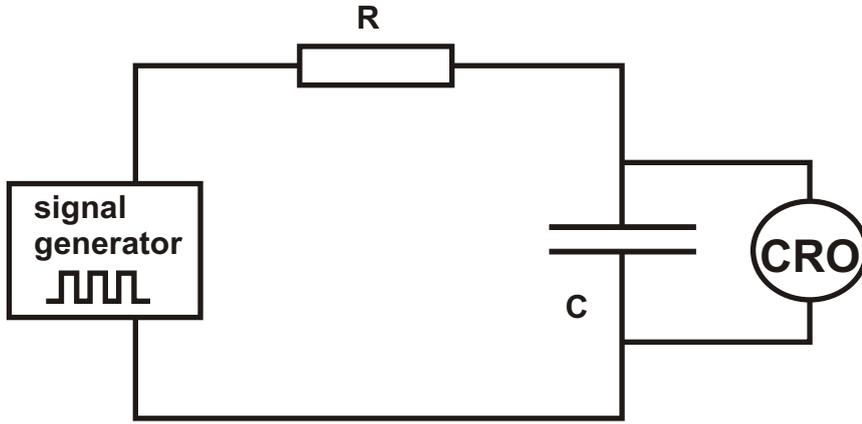
Capacitors do not affect the shape of a sine wave signal. However they do affect the shape of other types of signal like the square wave shown below.
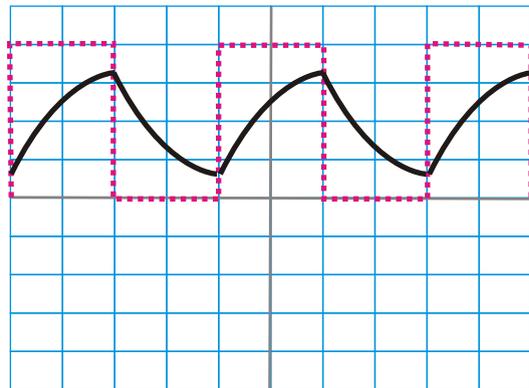The shape is affected by how fast the capacitor can charge or discharge.
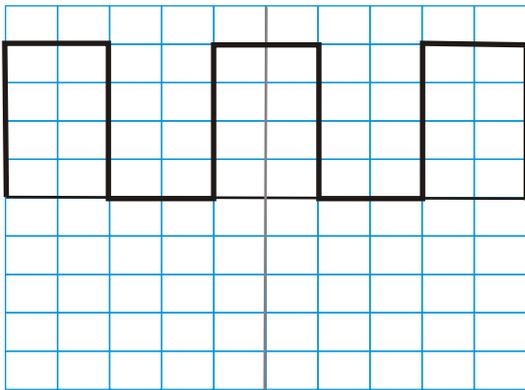
Input Signal
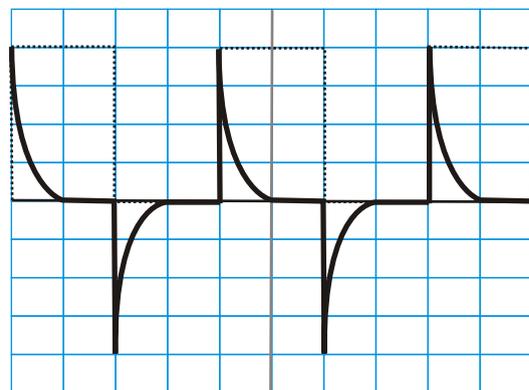

Small R + small C

Small R + Large C

Large R + small C

The change in the p.d. across the resistor R

The signal across the resistor R shows the changes in the current flowing in the circuit.
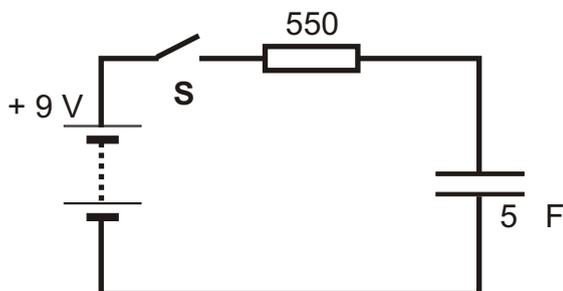
$$V_R = I \times R$$

constant

Signal across R : small R + small C

Example

The circuit shown in the diagram is set up. The battery has an e.m.f. of 9 V and negligible internal resistance. The capacitor is initially uncharged. Switch, S, is closed.

(a) Calculate the current flowing in the circuit immediately after the switch is closed.

(b) After a short time the p.d. across the capacitor is 3 V. Calculate the current flowing in the circuit.

(c) Calculate the charge stored in the capacitor when the capacitor is fully charged.

(a) *Immediately after the switch is closed, the p.d. across the capacitor is zero. The p.d. across the resistor is 9V. Current flowing in the resistor is given by*

$$I = \frac{V}{R}$$
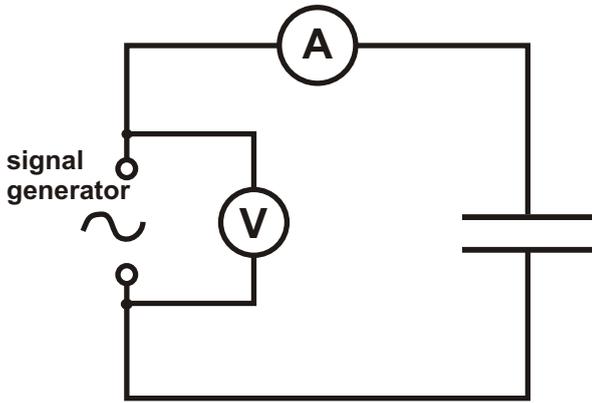
$$= \frac{9}{550}$$

$$= 0.016 \ A$$

$$= \underline{16 \ mA}$$

(b) *When the p.d. across the capacitor is 3 V, the p.d. across the resistor is 9 - 3 = 6 V.*

$$I = \frac{V}{R}$$

$$= \frac{6}{550}$$

$$= \underline{10 \ mA}$$

(c) *When fully charged, the p.d. across the capacitor is 9 V*

$$Q = C \ x \ V$$

$$= 10 \ x \ 10^{-6} \ x \ 9$$

$$= \underline{9 \ x \ 10^{-5} \ coulombs}$$

Capacitor current and frequency

A variable frequency a.c. supply is placed across a capacitor. The p.d. across the supply is kept constant and the frequency of the supply varied. The current flowing through the capacitor is measured using an a.c. ammeter.
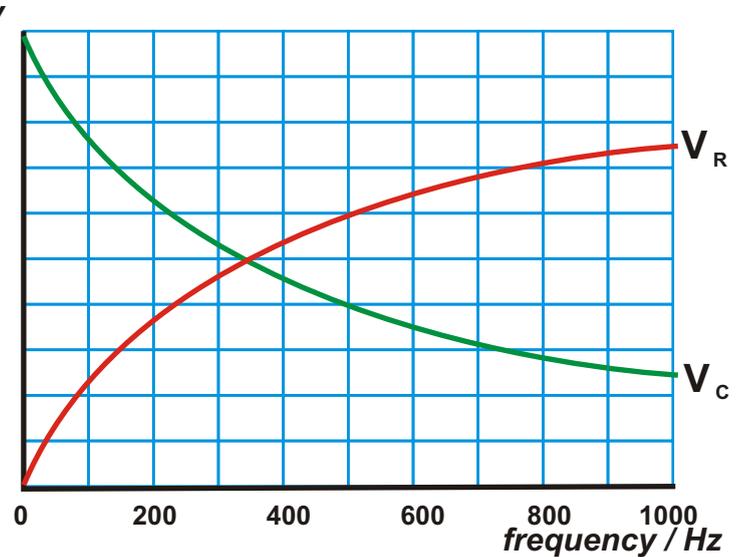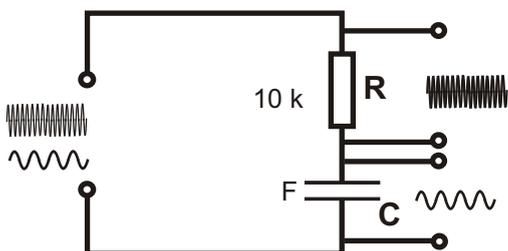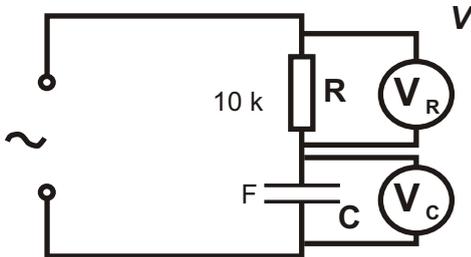
The current flowing in a capacitive circuit varies directly as the frequency of the source when the p.d. across the capacitor is kept constant.

$$I_C \propto f$$
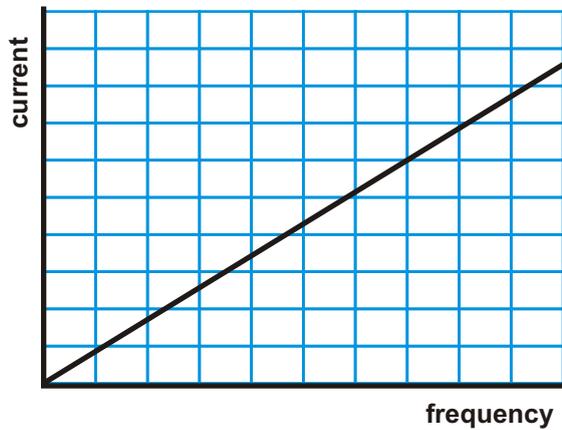
The resistance to a.c. current in a capacitor falls as the frequency of the a.c. increases.
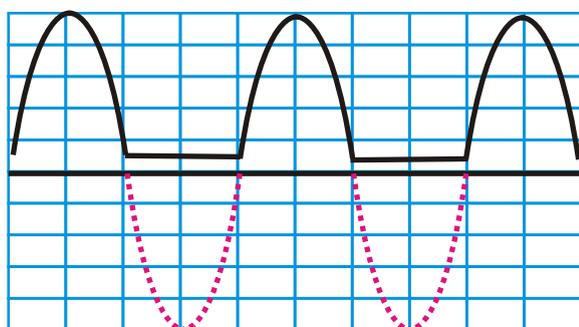
Filters

If the circuit above were supplied with a signal containing high and low frequency a.c., the part of the signal across the capacitor would contain most of the low frequency while that across the resistor, most of the high frequency.

The capacitor filters out the high frequency signal, the resistor the low frequency signal.

Smoothing
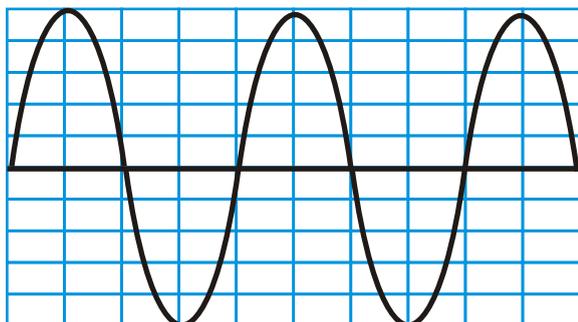
A.c supply voltage

Rectified a.c. voltage
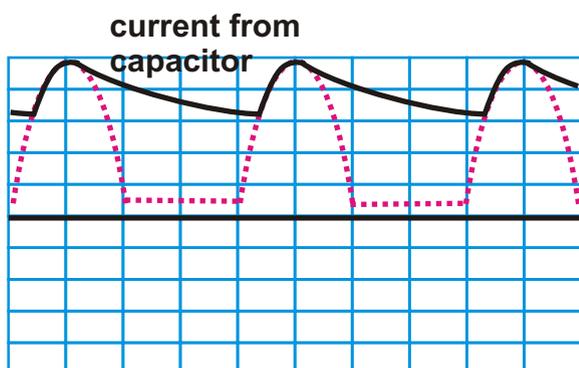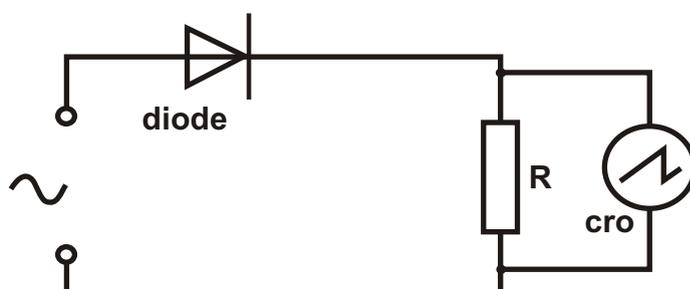
**Smoothed rectified voltage.
Charge from the capacitor provides
current to fill in the gap between the
peaks.**
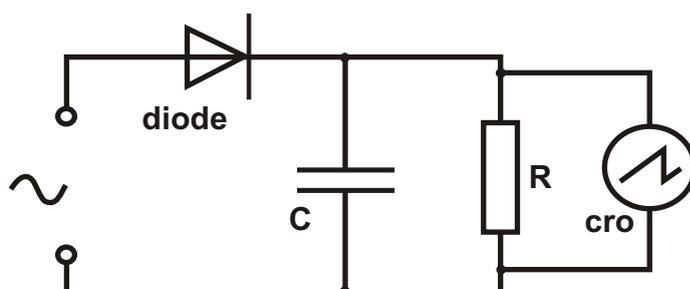
Capacitors are also used as supressors. Connected across the termonails of electric motors, they eliminate the sharp spikes in voltage which occur when the brushes switch segments on the motors commutator. These cause noise on radio and TV sets. The capacitor slows the build up of the spike.

Coupling capacitors.

d.c.

capacitor removes d.c. component of signal

0

0
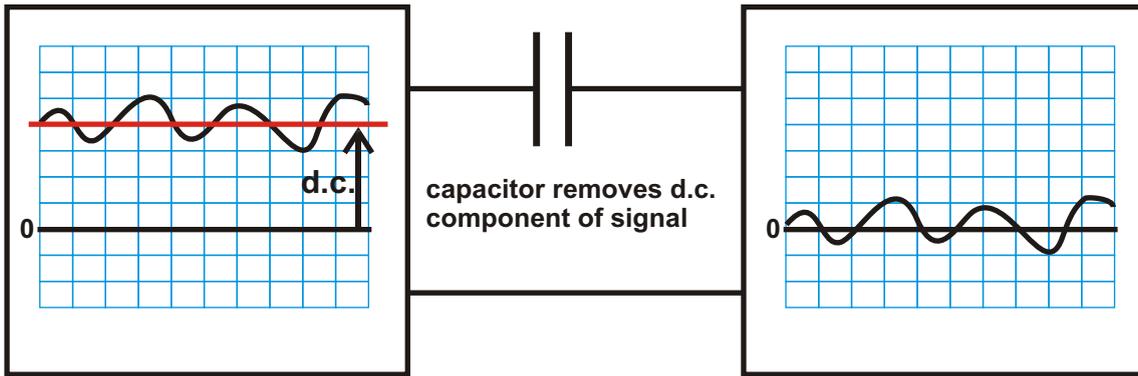
Capacitors block the flow of d.c. current in circuits, but allow a.c. current to flow. Capacitors are used to pass signals from one stage in a circuit to another where the d.c. added by one stage in the processing is removed

Capacitors in radio

Simple AM receiver

C2

C1

tuner   detector   AF amplifier
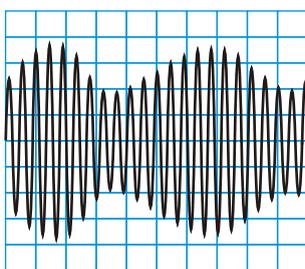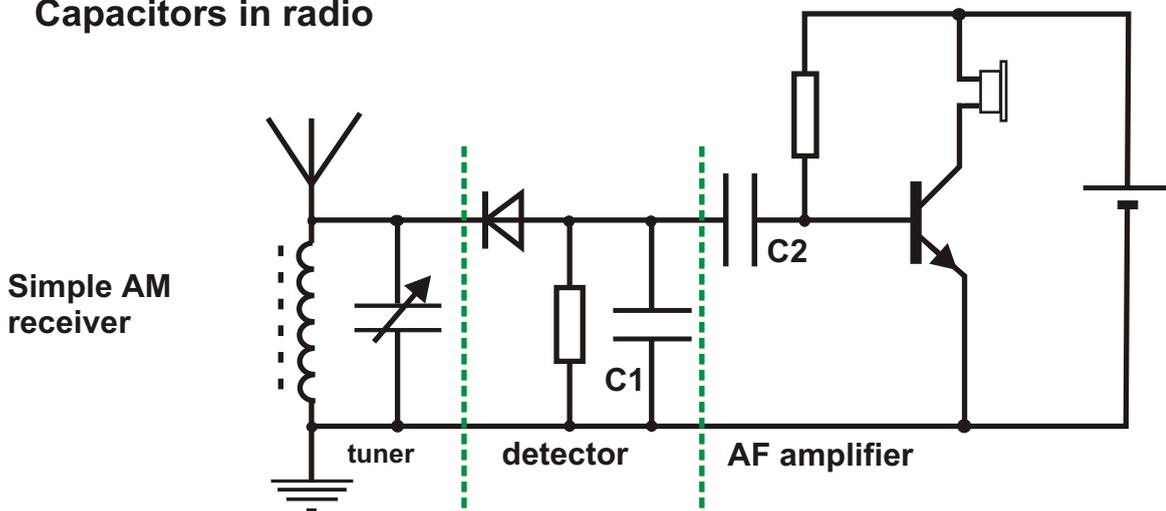
modulated carrier wave from tuner circuit
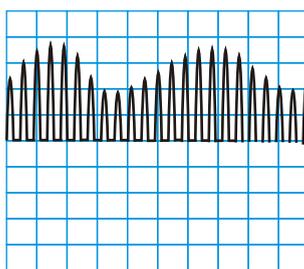
rectified carrier wave
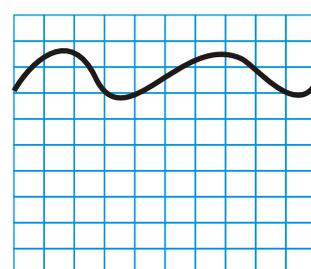
smoothed rectified carrier

C1 smooths rectified carrier wave to restore af signal

C2 removes d.c from signal.
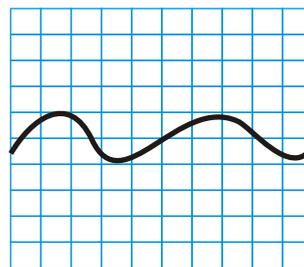
AF signal to amplifier
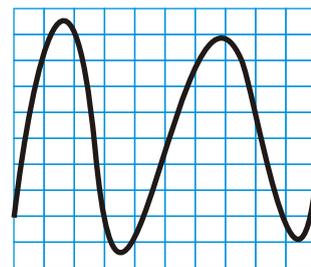
amplified AF for earpiece

Analogue Electronics

Operational Amplifier

An operational amplifier is a small integrated circuit which is frequently used in measuring instruments.

$+V_S$

$-$

V_1

$+$

V_2

$-V_S$

V_{OUT}

0 V
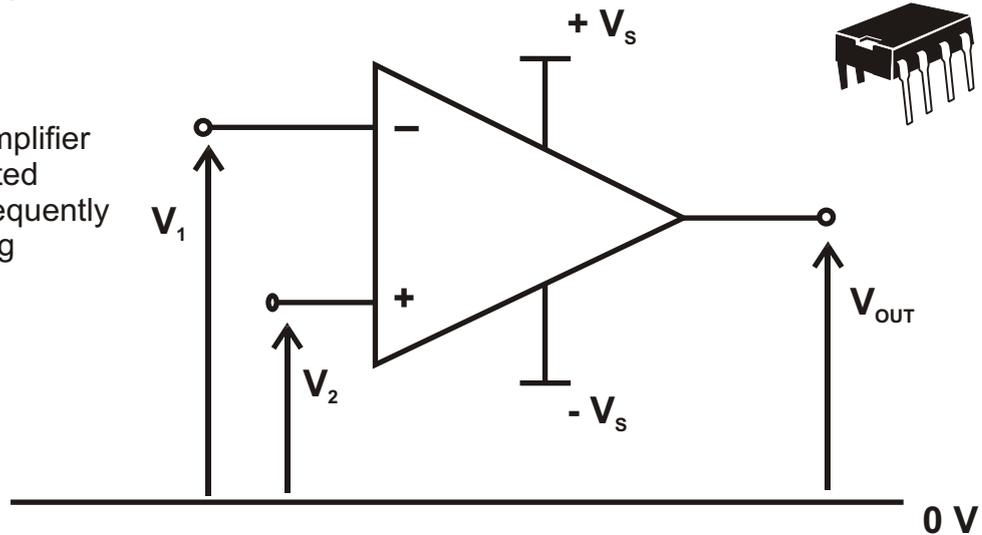
Basic relationship

$$V_{OUT} = A_O (V_2 - V_1)$$

Where A_O is the open loop gain, typically over 500,000 !

Operational amplifiers are designed to amplify voltage. The output current is limited to a maximum of a few milliamps. An op-amp cannot be used to drive a loudspeaker or an LED.

The basic op-amp, with its massive gain, is very unstable. The slightest stray voltage across the inputs will cause the output to change drastically. If the op-amp is going to be used as an amplifier, then the gain has to be reduced. This can be done in several ways, the simplest way is shown below.

R_f **feedback resistor**

input resistor
R_1

$+V_{IN}$

$-$

$+V_S$

$+$

$-V_S$

$-V_{OUT}$

0 V
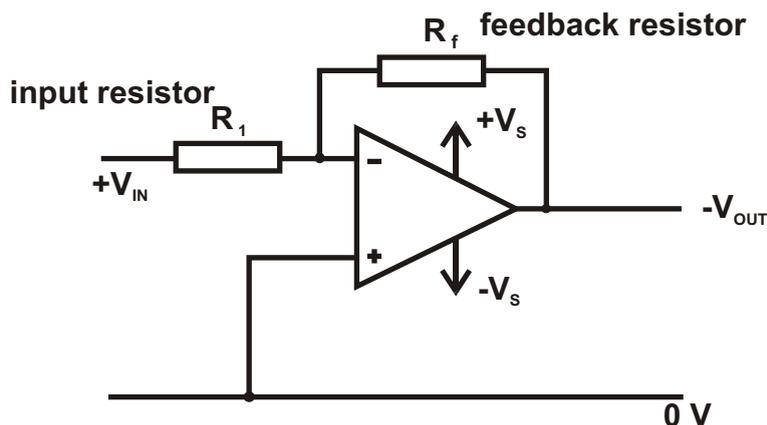
With the '+' input connected to 0 V, the equation above becomes $V_{OUT} = A (0 - V_{IN})$

$$= - A \, V_{IN}$$

If V_{IN} is positive, then V_{OUT} will be negative. Any voltage fed back to the input will reduce the input and reduce the gain. This set up is an op-amp used in the INVERTING MODE.

Gain of an op-amp in the inverting mode

The ideal op-amp has infinite gain, infinite input resistance and zero output resistance. It has no effect on any voltage it measures and its output can be passed on with no effect on the next stages.

For an ideal op-amp we can state.

1. *The input current flowing into the op-amp is zero (infinite resistance)*

2. *There is no potential difference between the inputs (infinite gain means that for any output voltage the corresponding voltage across the input terminals is small; virtually zero.)*
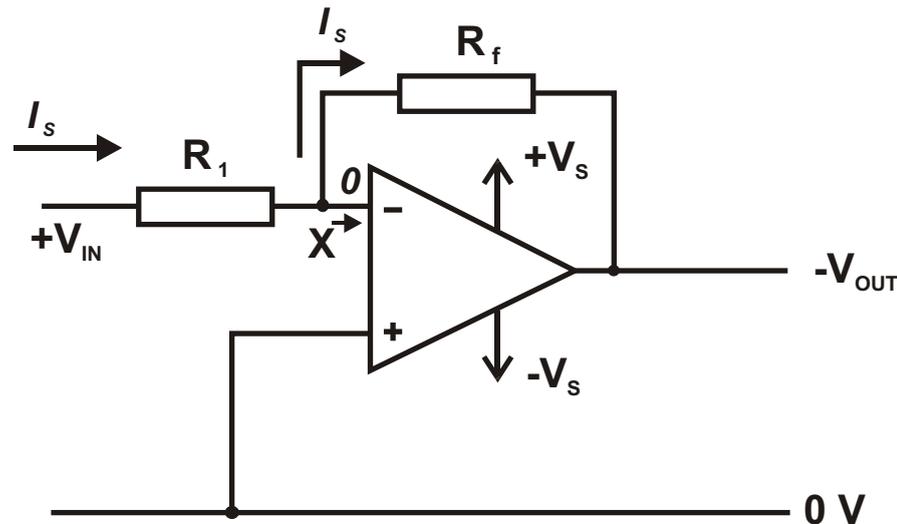
For an ideal op-amp, the voltage at point X is held at 0 V, the same voltage as the '+' input which is connected to the 0 V rail.

At junction X, no current will flow into the op-amp, so all the input current will flow through both resistors. We can use this to work out an expression for the gain.
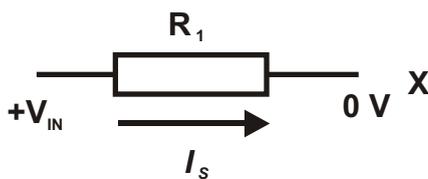
$$I_S = \frac{+V_{IN}}{R_1}$$
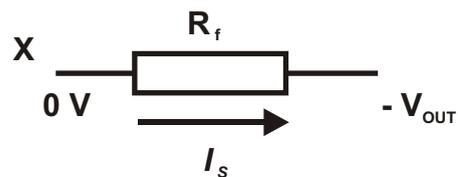
$$I_S = \frac{0 - V_{OUT}}{R_f} = \frac{-V_{OUT}}{R_f}$$

both currents are equal

$$\frac{-V_{OUT}}{R_f} = \frac{+V_{IN}}{R_1}$$

rearranging:

$$Gain = \frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_1}$$

For an ideal op-amp, the gain depends only on the ratio of two resistors.

Practical op-amp
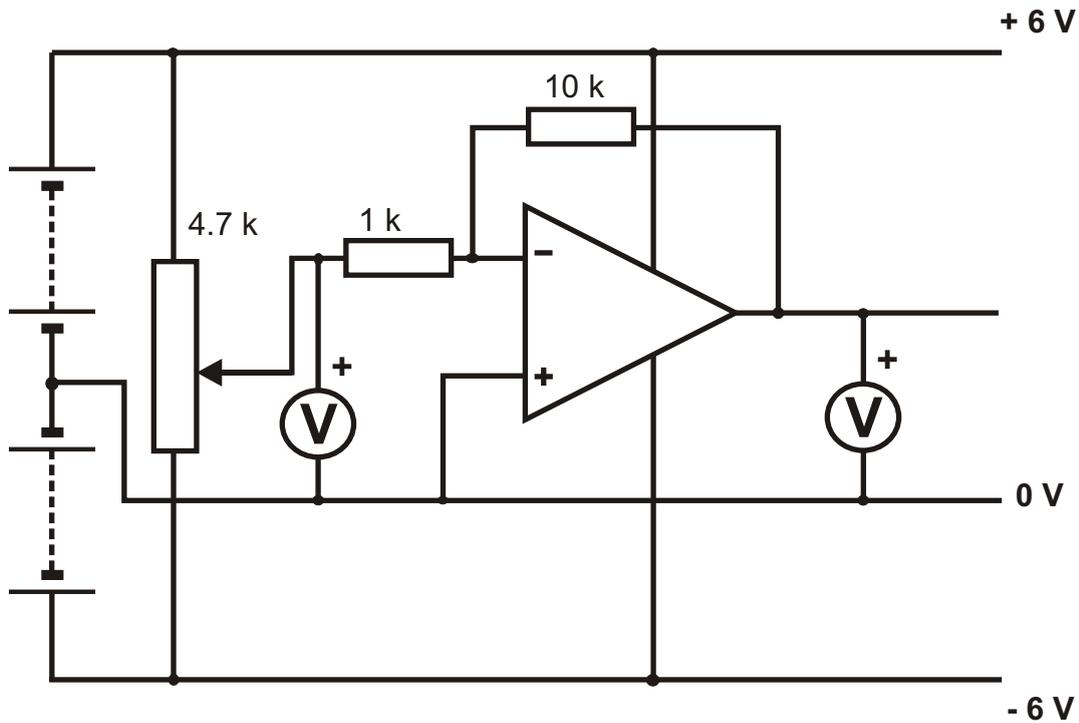
If we investigate the characteristics of a practical inverting mode amplifier, we obtain a graph like the one shown opposite.

gain = - 10

$$= -\frac{10\ k}{1\ k}$$

This is an identical result to the one we would expect for the ideal op-amp.

$$gain = \frac{V_{OUT}}{V_{IN}} = \frac{-6.0}{0.6}$$
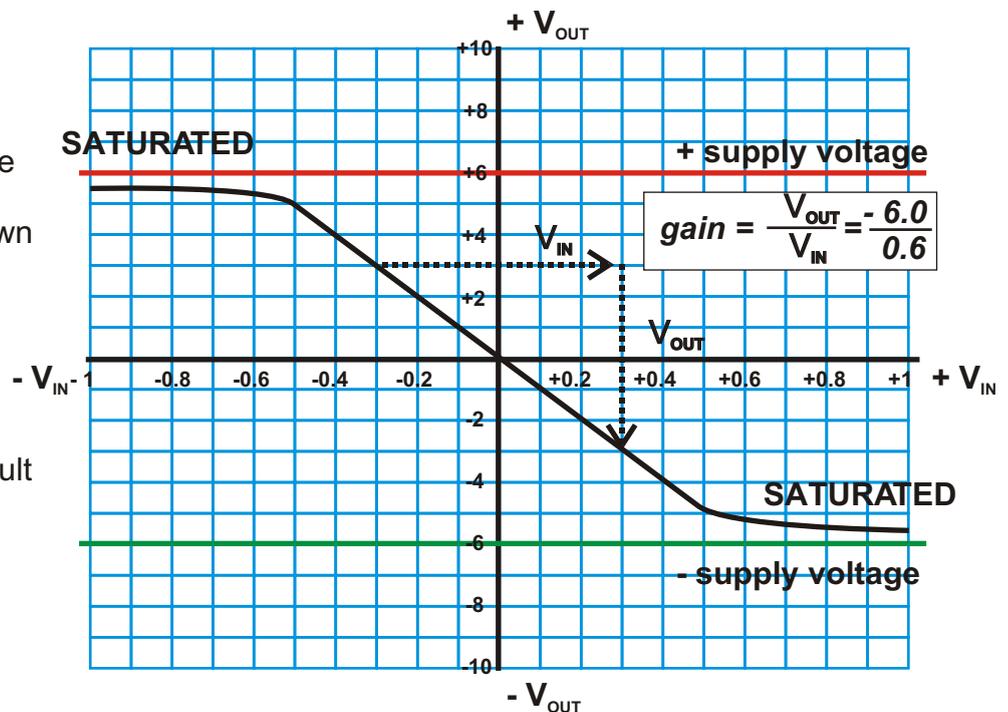
The output voltage of the op-amp is derived from the supply voltage. This means that the output voltage cannot rise above or fall below the supply voltage. In the situation where the output voltage has reached this limit, the op-amp is said to be *saturated.*
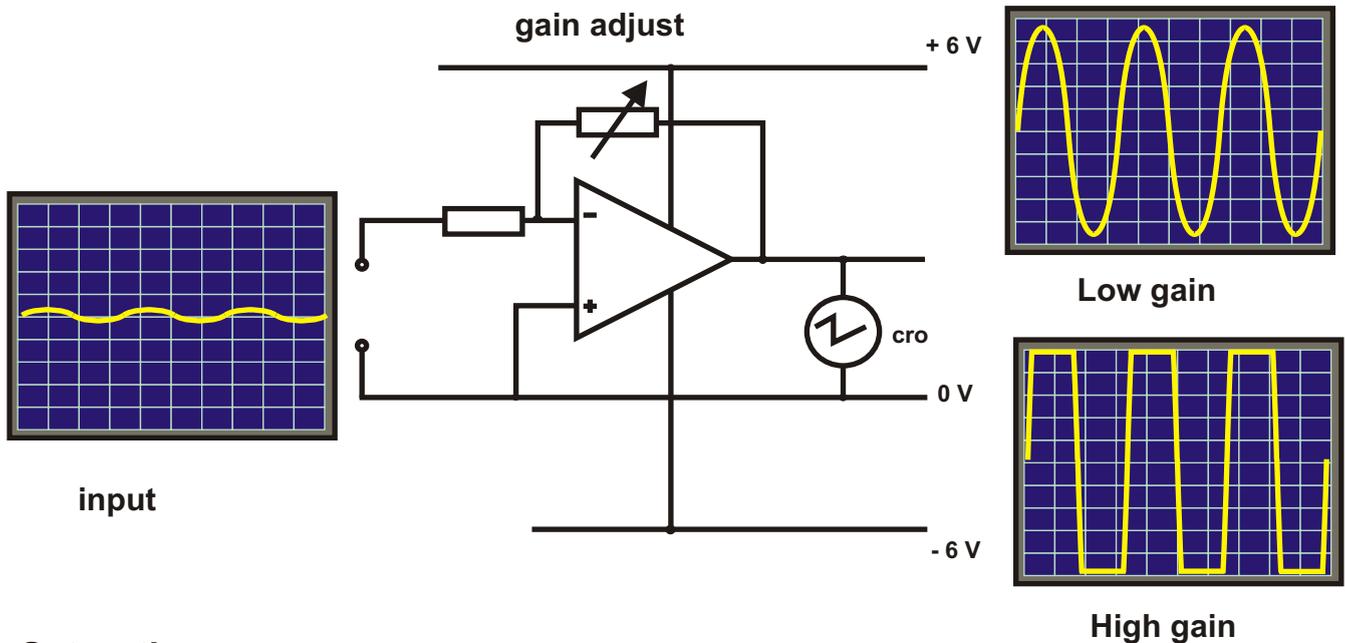
gain adjust

+ 6 V

input

Low gain

cro

0 V

- 6 V

High gain

Saturation

The output voltage cannot rise above the positive supply potential or fall below the negative supply potential.

When an a.c. signal is applied to the inputs of a suitable op-amp, a low gain will produce an an amplified a.c signal. Increasing the gain will drive the op-amp into saturation an the output signal will br 'clipped' close to the supply voltages upper and lower limits.

Example

(a) Find the output voltage when the input voltage is 0.6 V

(b) The input voltage is raised to 2.0 V. Explain what happens to the output voltage.

+ 9 V

100 k

10 k

V_{IN}

V_{OUT}

0 V

- 9 V
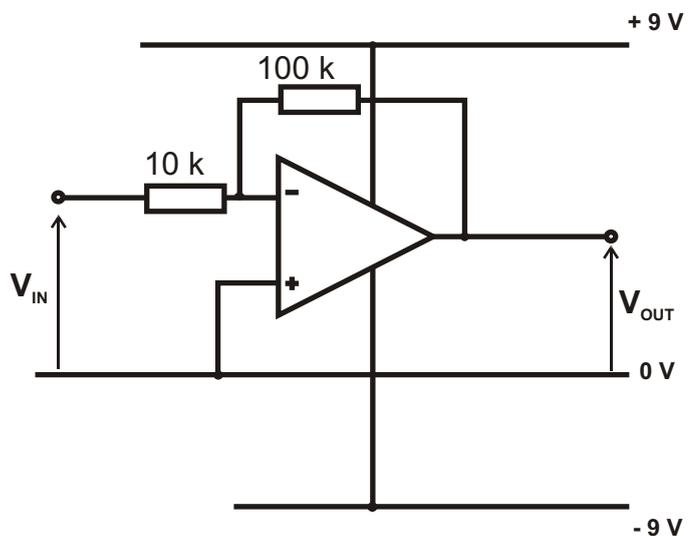
(a)

$$\frac{V_{OUT}}{V_{IN}} = - \frac{R_f}{R_1}$$

$$\frac{V_{OUT}}{0.6} = - \frac{100}{10}$$

$$\underline{V_{OUT} = - 6.0 \text{ V}}$$

(b) When the input voltage is raised to 2.0 Volts, the amplifier will go into saturation. The output voltage will be around the value of the negative supply, - 9 V.

Differential Mode
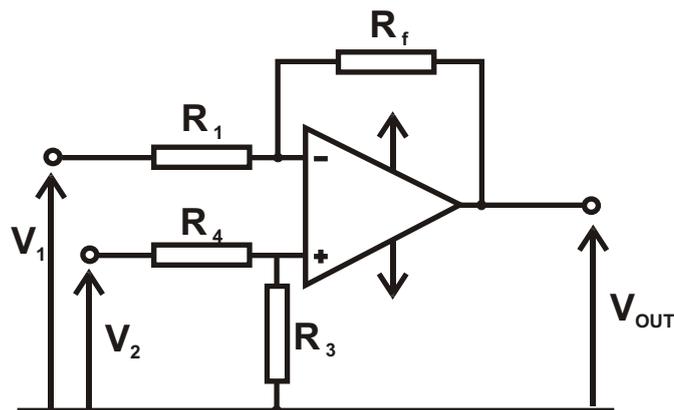
If $\dfrac{R_f}{R_1} = \dfrac{R_3}{R_4}$, the following relationship applies

$$V_{OUT} = (V_2 - V_1)\,\dfrac{R_f}{R_1}$$

This arrangement amplifies the difference between the input voltages. It is often referred to as a difference amplifier.

Example.
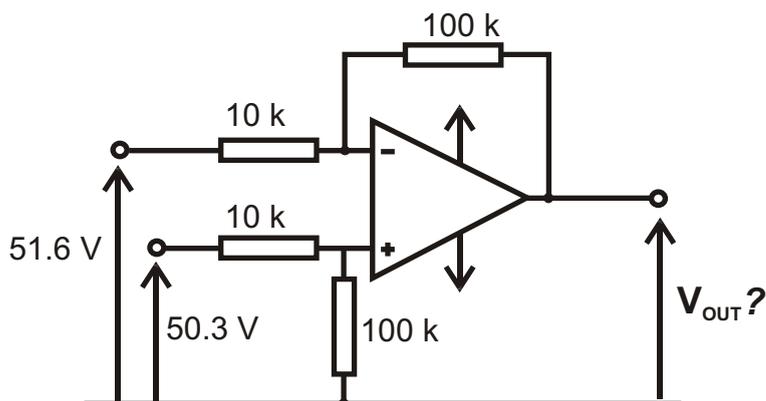
The ratio of the resistors is the same, so the relationship shown below applies.

$$V_{OUT} = (V_2 - V_1)\,\dfrac{R_f}{R_1}$$

$$= (50.3 - 51.6) \times 10$$

$$= -1.3 \times 10$$

$$\underline{= -13\ V}$$

The same restrictions apply: the output voltage cannot rise above the positive supply potential or fall below the negative supply potential.

Differential mode and the Wheatstone bridge.
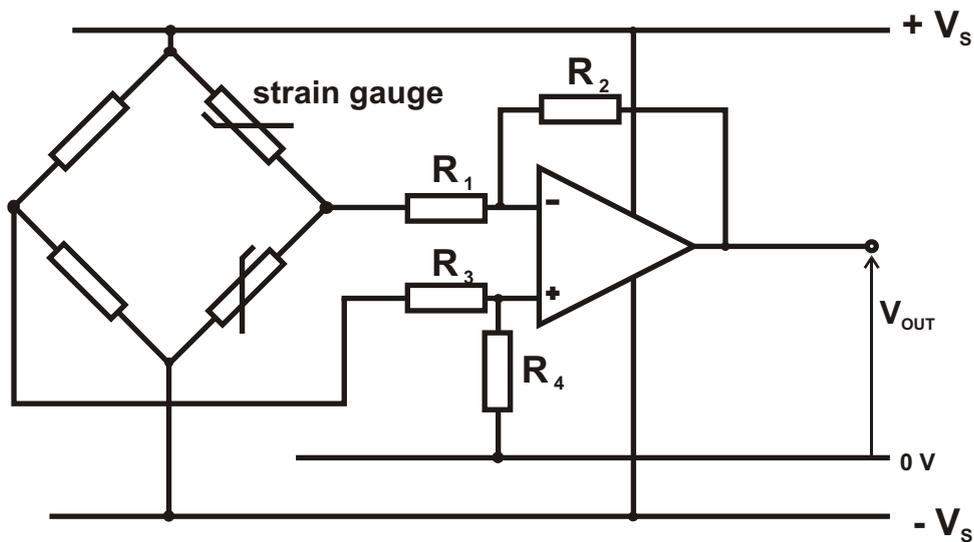
The op-amp, arranged in the differential mode, is used to replace the voltmeter in the Wheatstone Bridge. This allows the amplification of the small p.d.s generated by resistive sensors like strain gauges and resistance thermometers.

Sensors, like strain gauges are normally fitted as pairs to allow for temperature fluctuations. One of the strain gauges is the dummy, the other is the measuring sensor. Any resistance change due to temperature in one is compensated by the equal change in the other, keeping the bridge in balance.

Op-amps used to control external devices.

Op-amps are unable to provide large output currents and cannot be used to operate devices which require current of any size. However, the output current from the op-amp is enough to turn a transistor on and off and this can be used to provide a current.

Frost alarm
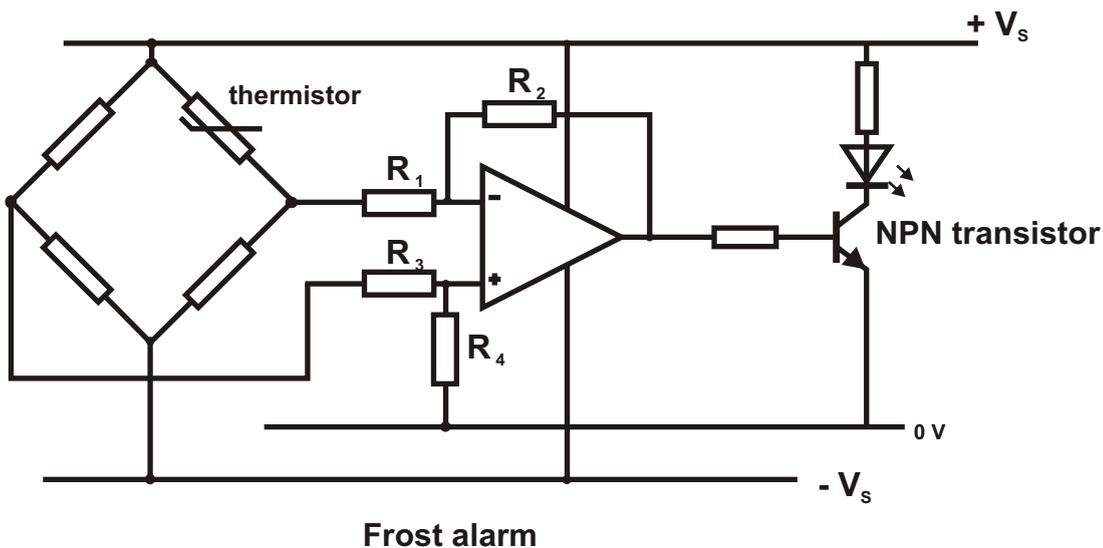
Higher Physics :   Radiation and Matter

Summary Notes

Waves

Frequency

Waves are created by vibrating sources and carry energy from one point to another. The frequency of a wave is equal to the frequency of the source which generated it. The frequency does not change when a wave passes from on medium into another.

medium 1   medium 2

$$v_1 = f \ x \qquad\qquad v = f \ x$$

The **period** of a wave is the time taken to complete one wave cycle of movement.

period

$$period = \frac{1}{frequency}$$

A wave with a frequency of 20 kHz, will have a period $= \dfrac{1}{20 \times 10^3}$ seconds

$$= 5 \times 10^{-5} \text{ seconds}$$

Transverse and longitudinal wave motion.

wave direction

particle movement

Transverse Waves : Particles affected by the wave move at right angles to the direction of movement of the wave. Light and water waves are transverse waves.

wave direction

particles are pushed together then pulled apart as wave passes

particle movement

Longitudinal Waves : Particles affected by longitudinal waves move in the same direction as the movement of the wave. Sound is a longitudinal wave.

Amplitude

The amplitude of a wave is a measure of the energy carried by the wave.

max E$_P$

max E$_K$

max E$_P$

position of
0 displacement
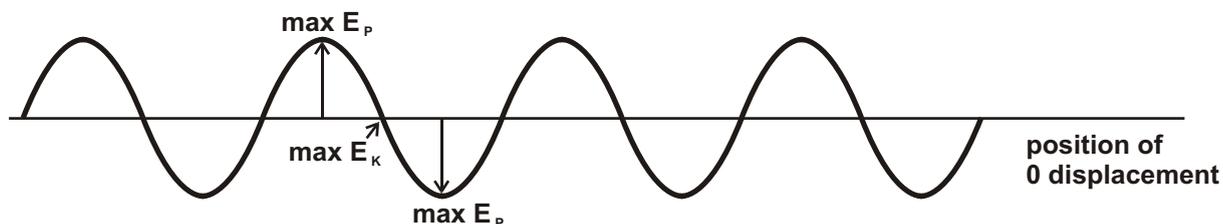
Water waves : Particles of water oscillate up and down as the wave passes. The amplitude represents the position of maximum potential energy.

Interference

Where two or more waves cross, the effect on a particle will be the sum of the separate wave effects. One wave interferes with the effect of the other.

For normal unrelated wave sources the effects will be complicated and difficult to predict. If the sources are **coherent**, however, it is possible to generate stable wave patterns or **interference patterns**.
Coherent waves are waves have equal frequencies and are generated in constant step with each other or, if not in step, they are out of step by a constant amount.

We can set up two coherent waves in a ripple tank using two dippers driven at the same frequency.
The result of this is a pattern of waves where in certain directions the waves have high amplitude and in between the waves have low amplitude.

Wave trains of large amplitude waves. Waves interfere constructively.

maxima

minima

low amplitude waves in between the high amplitude. The waves interfere destructively.
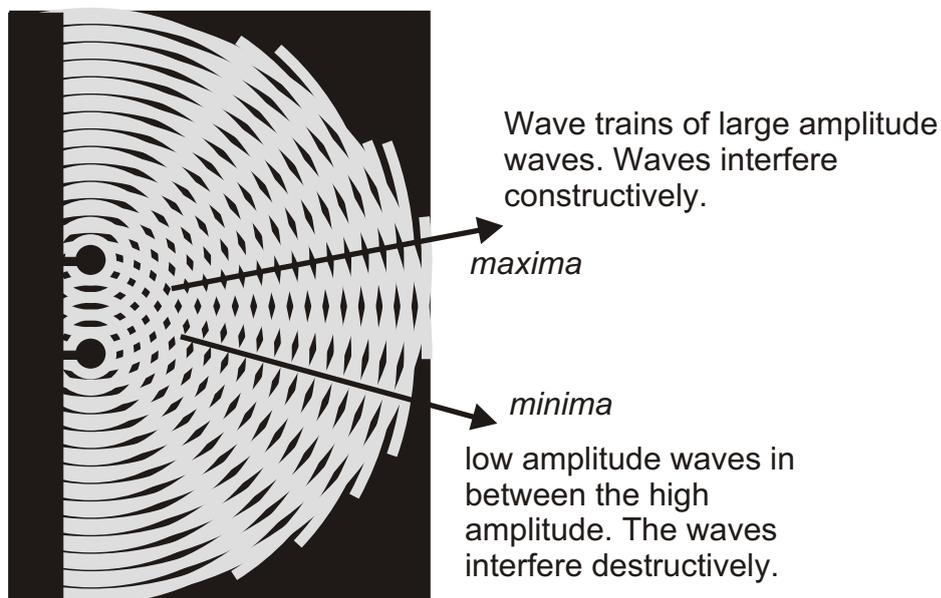
Constructive Interference

The troughs and peaks of one wave coincide with the troughs and peaks of the other. This produces a combined wave which has a greater amplitude.
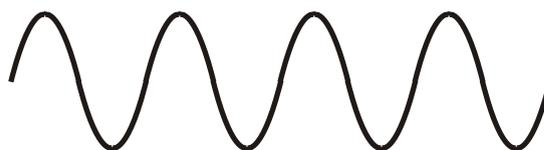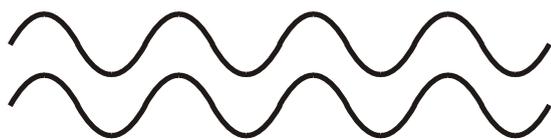
maxima

Destructive Interference

The peaks of one wave coincide with the troughs of the other. The waves 'cancel' producing a combined wave of low amplitude
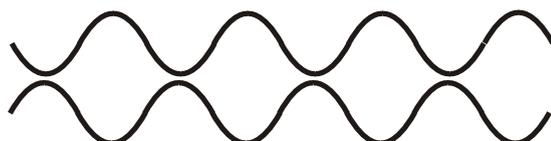
minima

Path Difference

maxima

d1

minima

S1

d2

S2
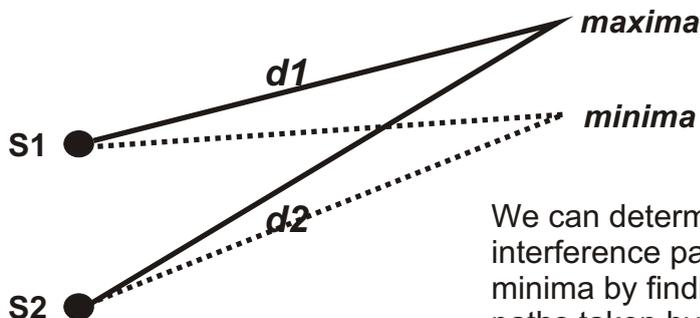
We can determine whether a point in an interference pattern is a maxima or a minima by finding the difference in the paths taken by the waves to reach the point.

Condition for a maxima (constructive interference)

$$\textit{Path difference} = \textit{d1 - d2} = \textit{n} \quad (\textit{n} = 0, 1, 2, 3, 4........)$$

Condition for a minima (destructive interference)

$$\textit{Path difference} = \textit{d1 - d2} = (\textit{n} + \tfrac{1}{2})$$

Interference of sound.

We can set up two loudspeakers and drive them from the same signal source as shown.
An observer walking from X to Y will hear alternate changes in volume from quiet to loud as he crosses the paths of the maxima and minima of the interference pattern.
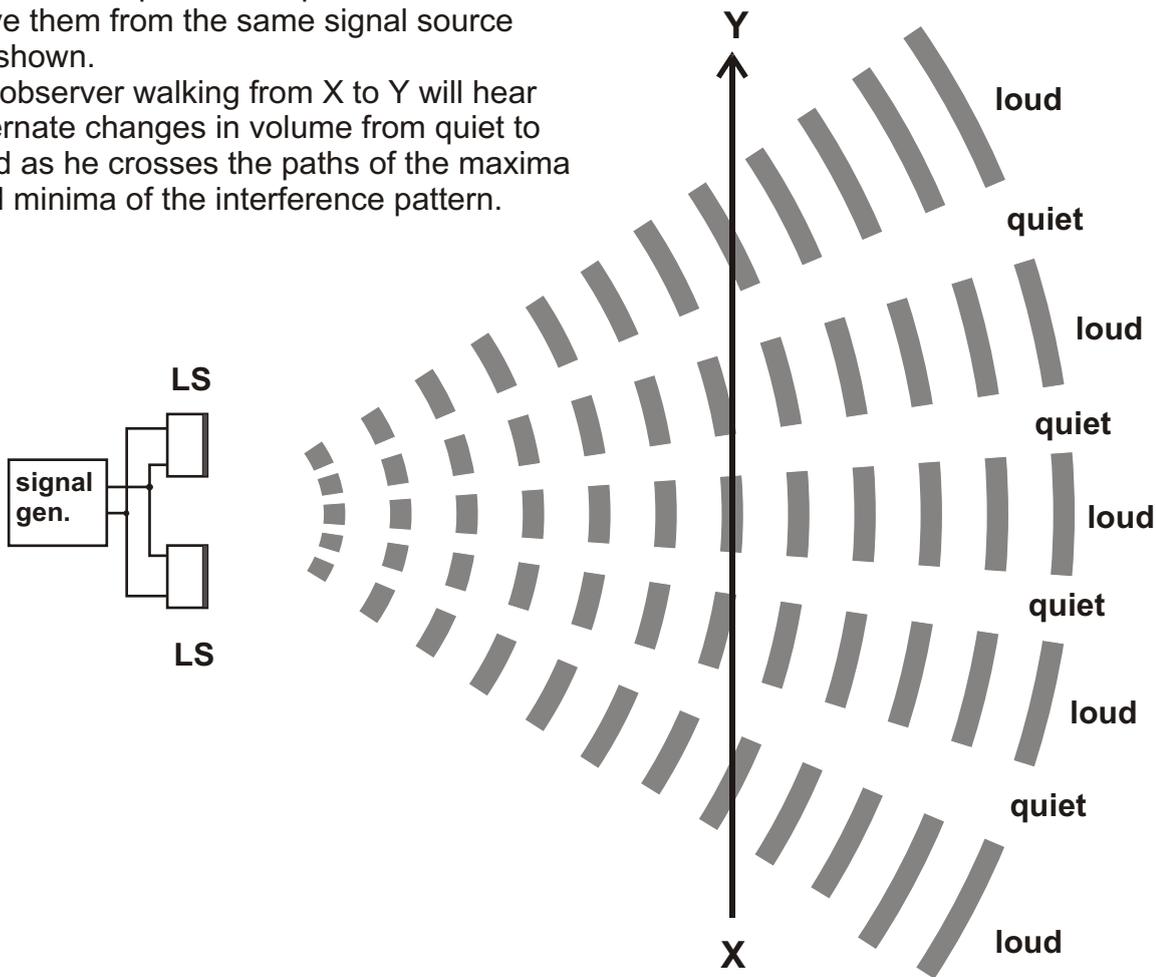
Example:
An interference pattern is set up using two loudspeakers. Maxima are located at A, B and C. Find the frequency of the sound.

Path difference at B = 0   (straight ahead)

Path difference at C =
                = 3.7 - 3.2
                = 0.5 m

$v = f x$

$340 = f x 0.5$ (speed of sound in air = 340 ms^{-1})

$f = 680\ Hz$
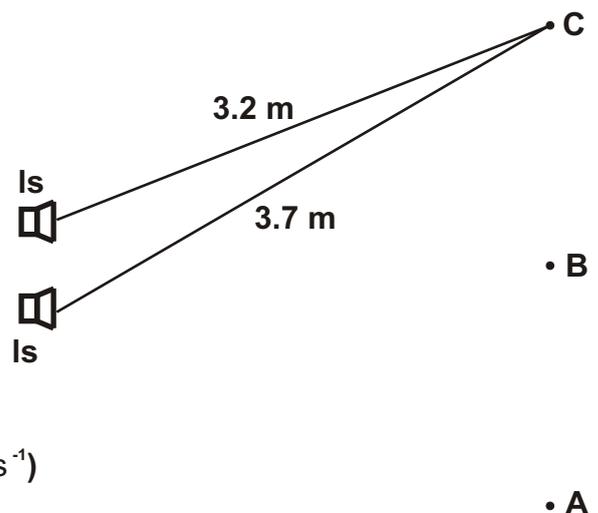
Double slit interference patterns.
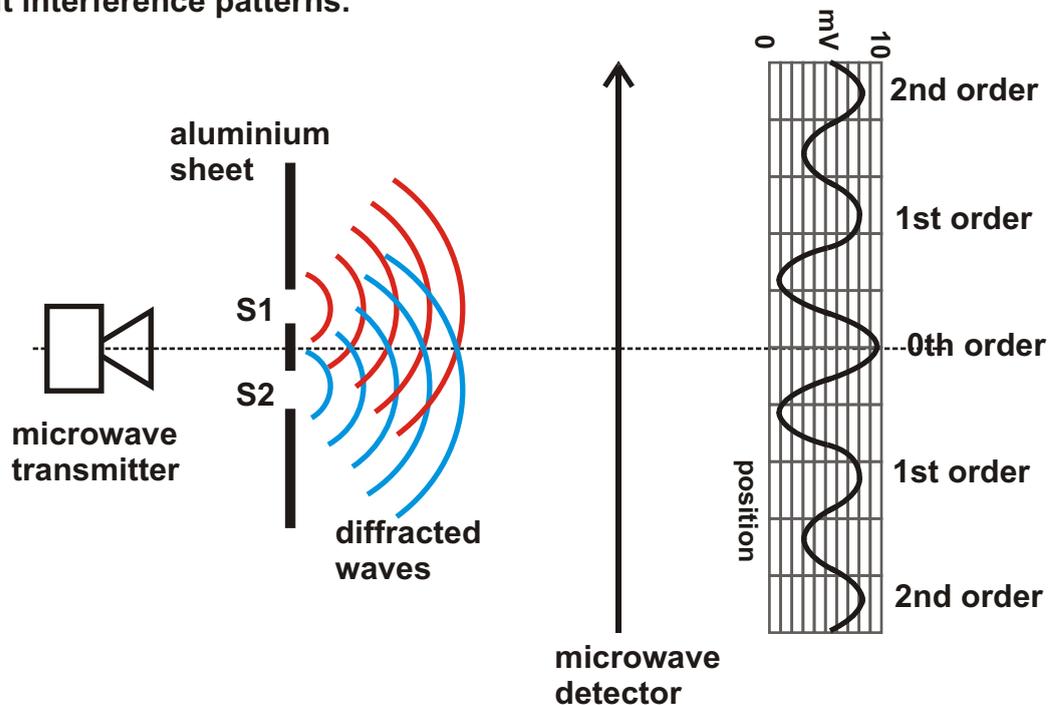
When microwaves are directed at a narrow gap between two sheets of metal, the microwaves beyond the gap are diffracted into a circular wave pattern. The gap behaves as a source of microwaves. With two gaps or 'slits', two wave patterns are produced. These are generated from the same source of microwaves so the wave patterns are coherent and will form an interference pattern.

This pattern can be detected using a special aerial connected to a voltmeter. A typical response is shown in the graph.

Young's Slits

In a famous experiment in 1801, Thomas Young demonstrated the wave nature of light. He used double slit interference to show that light could form an interference pattern. The formation of an interference pattern is taken as the test for wave motion. Waves can interfere and cancel each other out. where particles simply accumulate!!
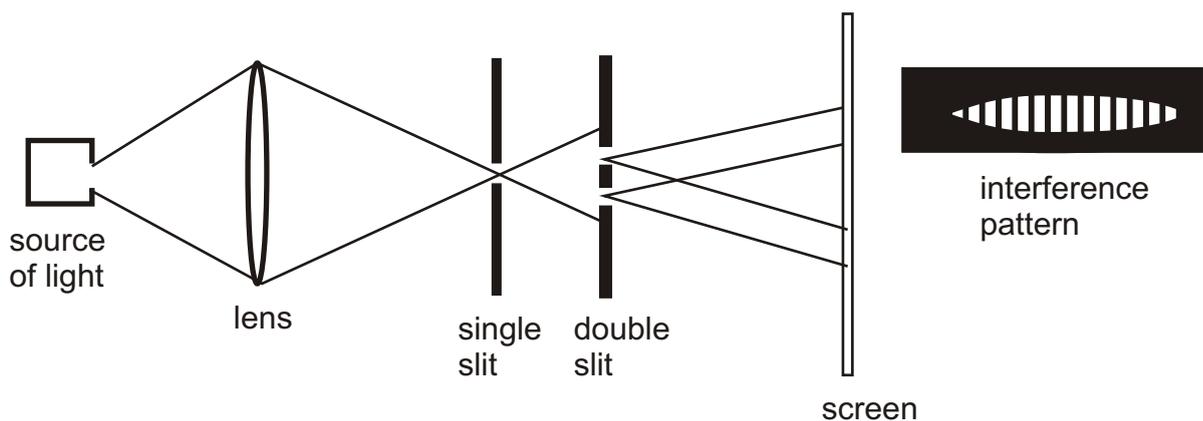
Young used his apparatus to measure the wavelength of light and convince the physicists of the nineteenth century that light was a form of wave motion.

The diffraction grating.

A diffraction grating consists of a piece of plain glass or plastic sheet with parallel slits engraved or photographed onto one surface.

The slits act as sources of diffracted waves. When illuminated from behind a series of closely spaced coherent sources are produced. These are ideal generating interference patterns. The diffraction grating produces bright maxima separated by dark

glass

'slits' engraved on glass surface

direction of maxima

d m

n

direction to 0th order
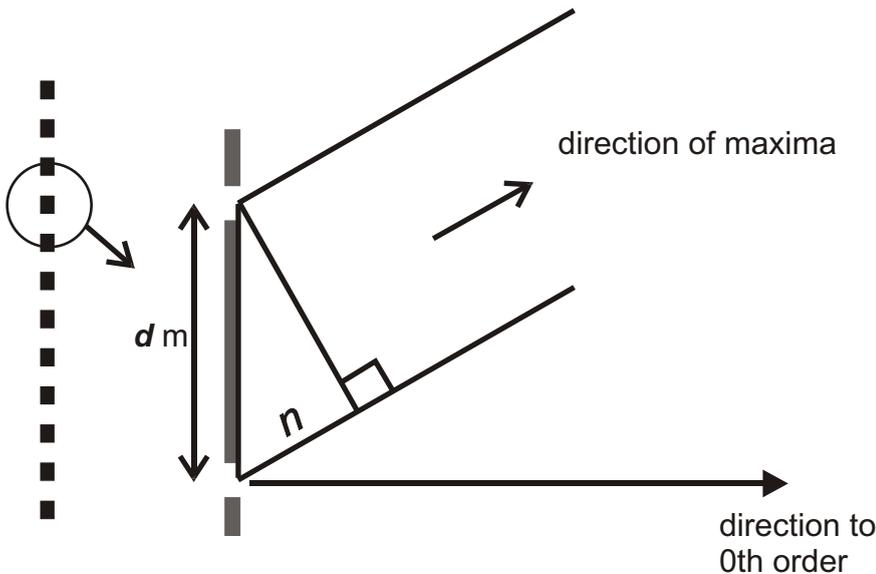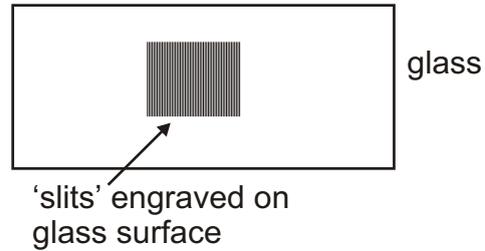
To produce a maxima in a given direction the waves from each slit must lag behind or in front of each other by an integral number of wavelengths.
From the digram, this occurs where:

$$n\lambda = d\sin$$

d is the distance between each slit in metres
is the angle between the direction of the maxima and the direction of the 0th order.
is the wavelength of the light in metres.

Measuring the wavelength of monochromatic light

Monochromatic light is light with a single wavelength. A laser produces a beam of coherent monochromatic light.

laser

D

X

3rd order n = 3

2nd order n = 2

1st order n = 1

0th order n = 0
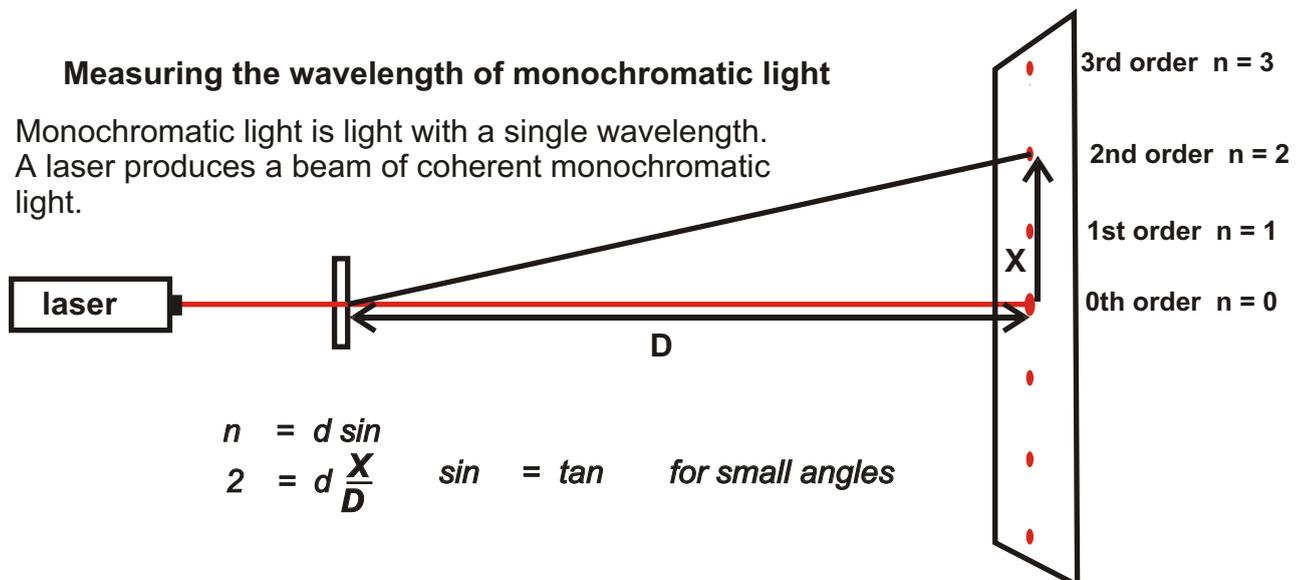
$$n\lambda = d\sin$$
$$2\lambda = d\frac{X}{D} \qquad \sin = \tan \quad \text{for small angles}$$

Spectrometer

slit

telescope

0th order

collimator

diffraction grating

turntable

light source

crosshairs

view through telescope
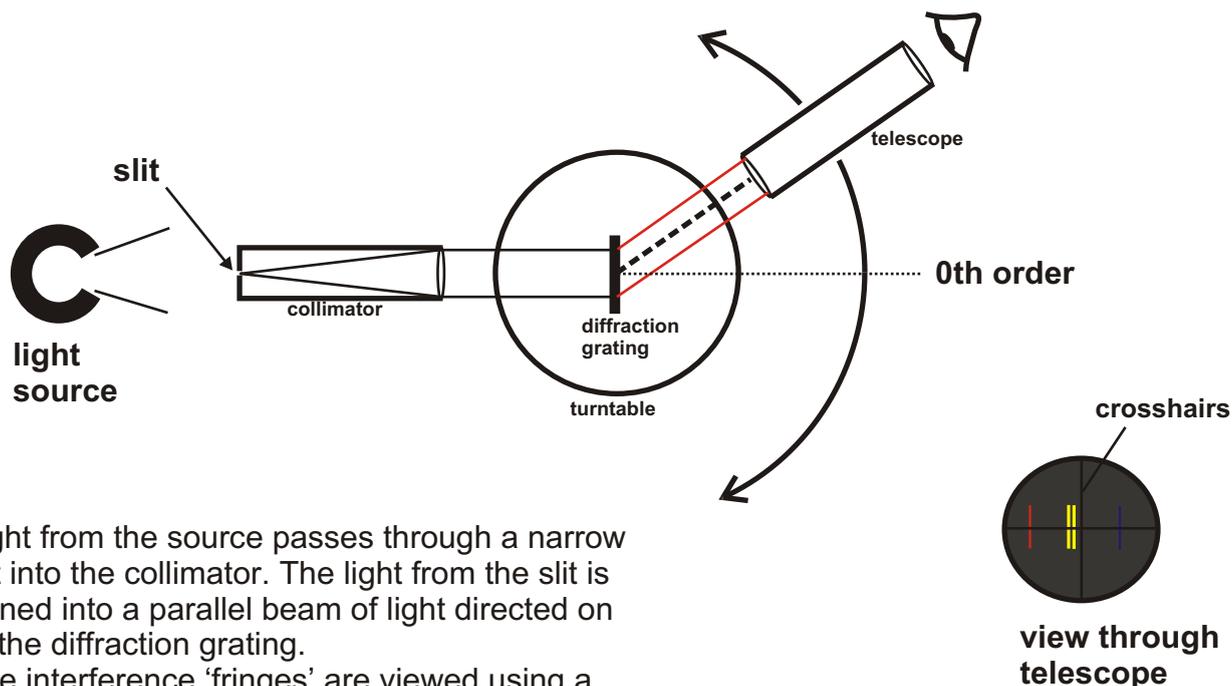
Light from the source passes through a narrow slit into the collimator. The light from the slit is turned into a parallel beam of light directed on to the diffraction grating.

The interference 'fringes' are viewed using a telescope (focussed on infinity). The observer sees diffracted images of the slit. The viewing angle is measured on the turntable.

Example: Monochromatic light is passed through a diffraction grating with a slit separation of 2×10^{-6} m. The first order maximum is viewed at an angle of $15.5°$.
Calculate the wavelength of the light.

$\lambda = d \sin \theta$

$= 2 \times 10^{-6} \times \sin 15.5$

$= 534$ nm

$d = 2 \times 10^{-6}$ m

$\theta = 15.5°$

White light spectra

diffraction
grating

white light

r

b

2nd order

1st order

0th order

1st order

2nd order
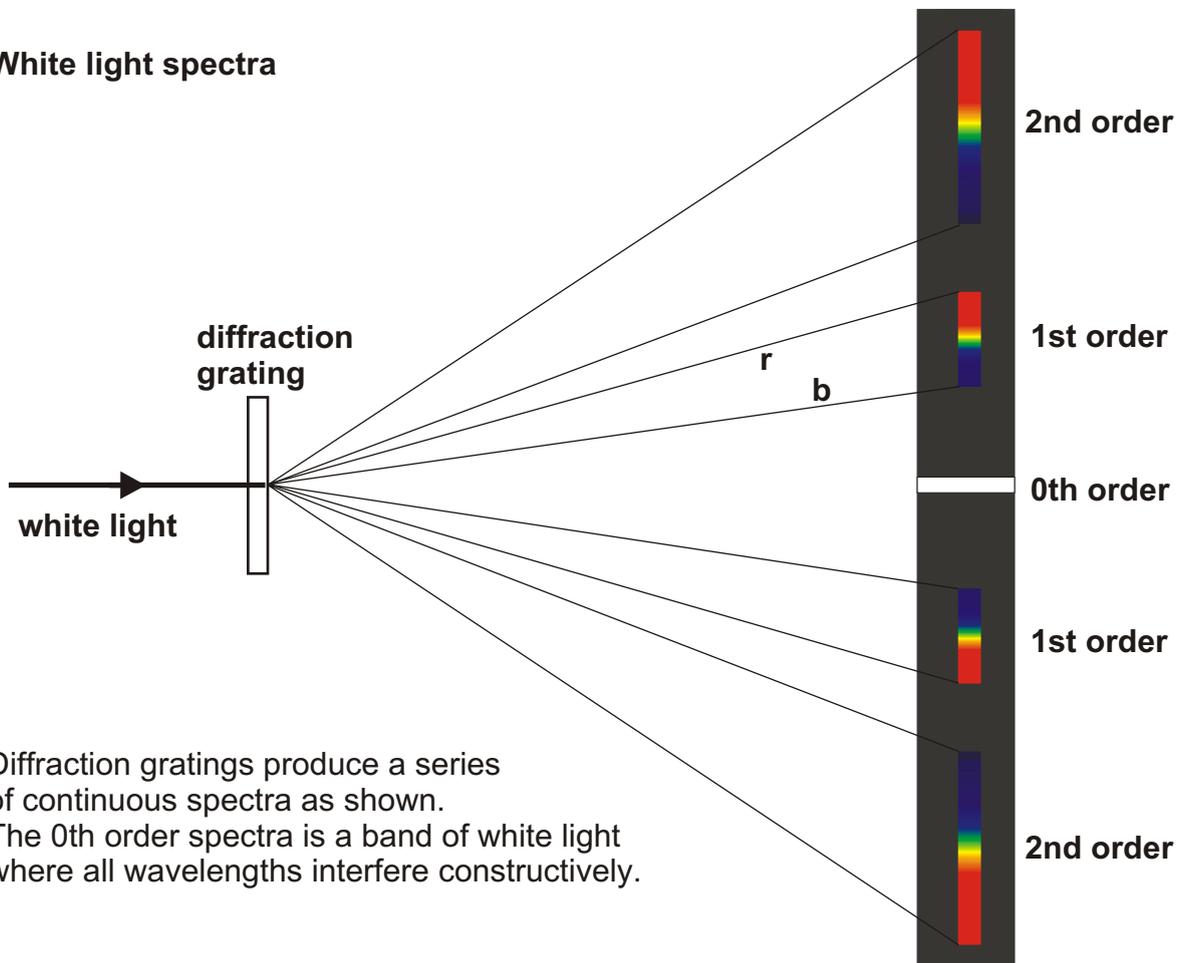
Diffraction gratings produce a series
of continuous spectra as shown.
The 0th order spectra is a band of white light
where all wavelengths interfere constructively.

Prism

white light

r

b
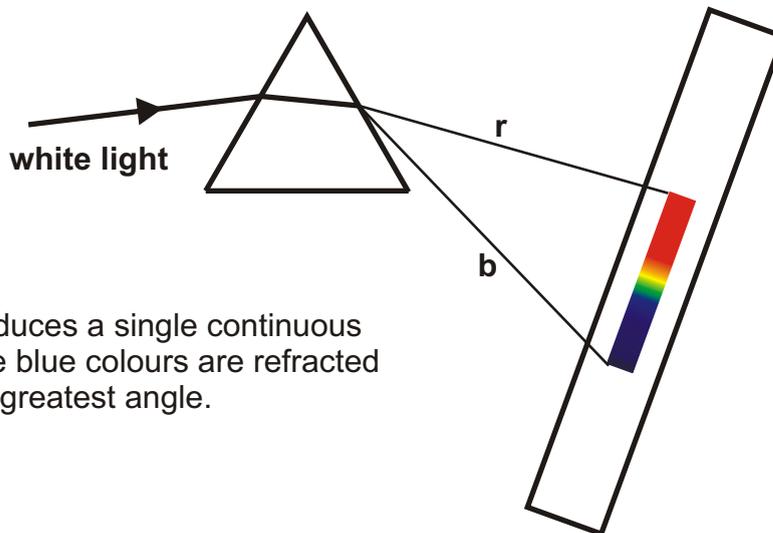
A prism produces a single continuous
spectra. The blue colours are refracted
through the greatest angle.

Wavelength and colour.

The wavelengths of light in the visible spectrum range from 700 nm at the red end to 400 nm at
the blue end. Green light is around 550 nm, the half-way point in the visible spectrum.

Refraction of Light
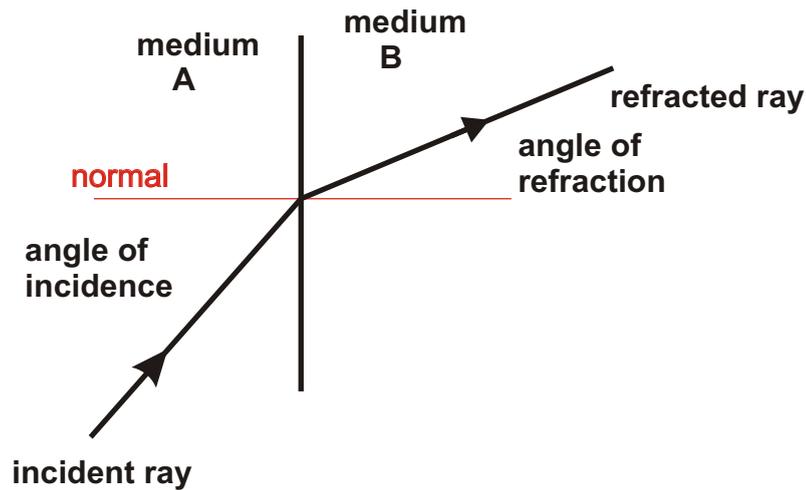
The speed of light depends on the medium through which it is travelling. When light passes from one medium to another, the change in speed causes the light to change direction. This change of direction at the boundary between two media is called refraction.
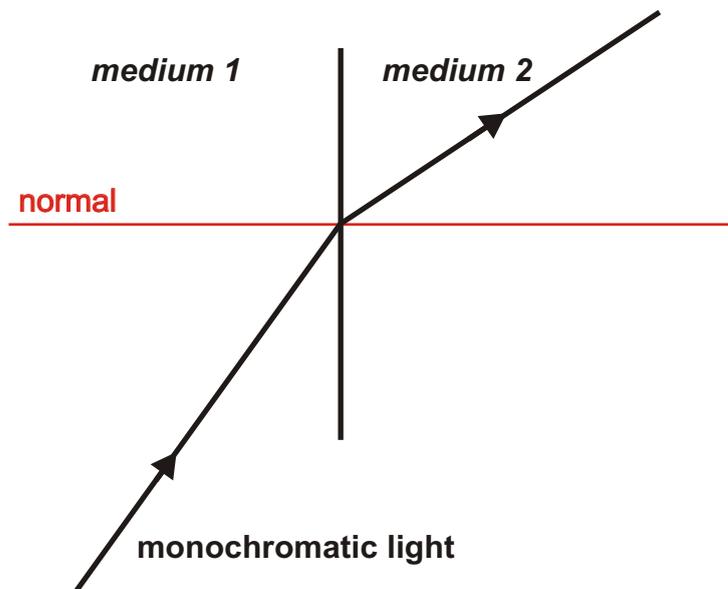
$$\frac{sin}{sin} \ = \ constant = refractive\ index$$

Note: The refractive index of optical materials depends on the frequency of the light involved. Normally examples will refer to *monochromatic light*. This is light with a single wavelength. Sources of monochromatic light include lasers.

The absolute refractive index.

The absolute refractive index (*n*) of an optical material is based on *monochromatic* light passing from a vacuum into the material

Vacuum

normal   M

V

Material

$$n = \frac{sin \; V}{sin \; M}$$

monochromatic light
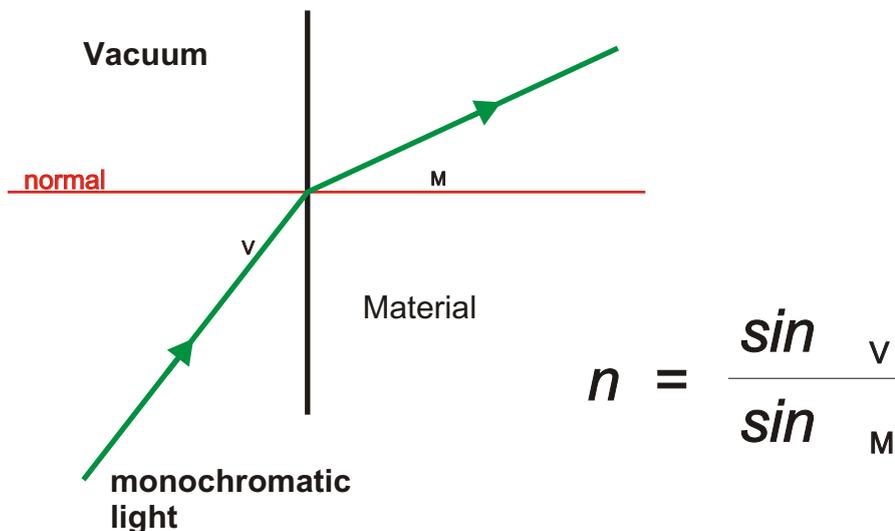
Measurement of refractive index

The speed of light in air is approximately equal to the speed of light in a vacuum so we can measure the refractive index using light from air into the material and be confident of accuracy to the fourth significant figure. This is more accurate than normal laboratory techniques will obtain.

ray from ray box directed at centre of face

refracted ray emerges undeviated

A narrow ray of light is projected at the centre of the flat face of a semi-circular block. The incident ray and the refracted ray are marked. The normal is drawn. Angles   and   ₂ are measured and the refractive index found from.

$$n = \frac{sin}{sin}$$

semi-circular block of material

monochromatic light
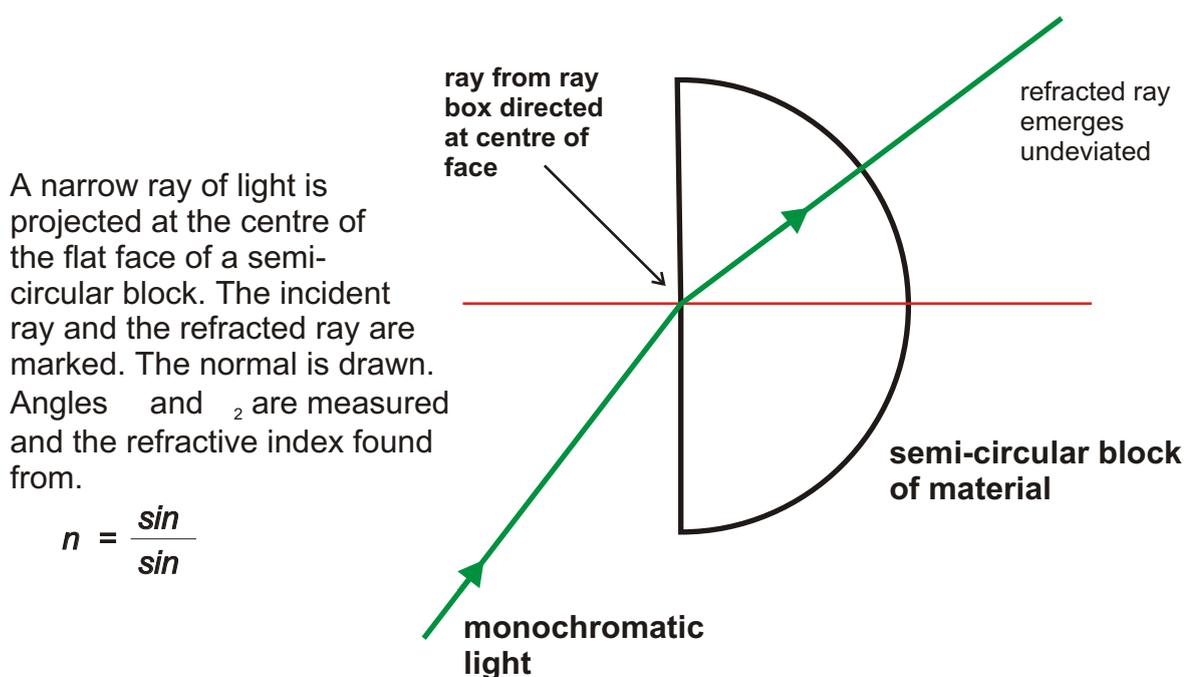
Material	Absolute Refractive index
Water	1.33
Ice	1.31
Perspex	1.49
Crown glass	1.50
Diamond	2.42
Air	1.00
Glycerol	1.47

The absolute refractive index is a ratio and so has no units. It is simply a number.

Example: Find the angle

$$n = \frac{\sin 50}{\sin}$$

$$\sin = \frac{\sin 50}{n}$$

n for air into water = 1.33

$$\sin = 0.576$$

$$= 35°$$

monochromatic light

air
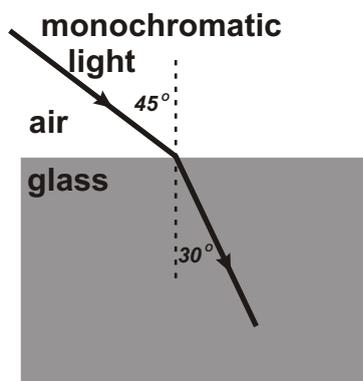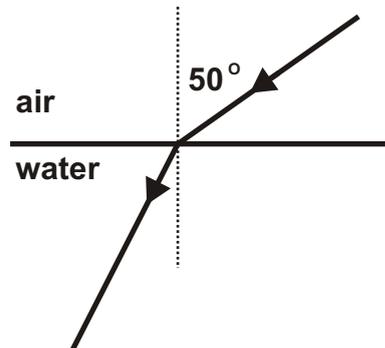
glass

Find the absolute refractive index of the glass in the diagram.

$$n = \frac{\sin 45}{\sin 30}$$

$$= 1.41$$

This value is approximately equal to the absolute value because the speed of light in air is close to the speed of light in a vacuum.

absolute refractive index =1.41

Critical Angle and Total Internal Reflection

When *monochromatic* light passes from an optically dense medium to an optically less dense medium light is directed away from the normal. As the angle of incidence increases the angle of refraction will eventually reach 90°. **Any increase in the angle of incidence past this point will cause the loss of the refracted ray and Total Internal Reflection. The angle of incidence which gives rise to an angle of refraction equal to 90° is** called the critical angle.

monochromatic light

less than the critical angle. Refracted ray with weak reflected ray,

equal to the critical angle. Weak refracted ray along flat face. Strong reflected ray.

greater than the critical angle. Total internal reflection with no refracted ray.

$$n = \frac{\sin 90}{\sin c}$$

$$= \frac{1}{\sin c}$$

vacuum

c

material

normal — A — v_2 — wave fronts — refracted ray — normal — C — D — B — incident ray — v_1

angle of incidence
angle of refraction
$_1$ = wavelength in 1
$_2$ = wavelength in 2
v_1 = speed in 1
v_2 = speed in 2

A ray of light is refracted as shown above. We can chose a section of that ray where an incident wave front is about to enter medium 2 at B and a refracted wave front is just leaving A.

Using trigonometry:

$$\frac{_1}{AB} = \sin \qquad AB = \frac{_1}{\sin}$$

$$\frac{_2}{AB} = \sin \qquad AB = \frac{_2}{\sin{}_2}$$

$$\frac{_1}{\sin} = \frac{_2}{\sin{}_2}$$

$$\text{refractive index } n = \frac{\sin}{\sin{}_2} = \frac{_1}{_2} = \frac{v_1}{v_2} \qquad (= v/f\,)$$

The refractive index of light passing from one material to another is the ratio of either the wavelengths in the materials or the speeds in the materials.

Example :   Find the speed of monochromatic light in glass with an absolute refractive index of 1.46.

$$n = \frac{c}{v}$$

c speed of light in vacuum = 3×10^8 ms^{-1}

v speed of light in glass

$$1.46 = \frac{3 \times 10^8}{v}$$

$$v = \frac{3 \times 10^8}{1.46}$$

$$= 2.05 \times 10^8\, ms^{-1}$$

Dispersion

The refractive index of an optical medium depends on the frequency of the light involved.. White light contains light with a range in frequencies. When refracted, the different frequencies take different paths through the medium. The white light is dispersed; spread out, by refraction.

blue light has a higher refractive index than red light in optical materials.

white light

glass prism

red (n = 1.505)

blue (n = 1.518)
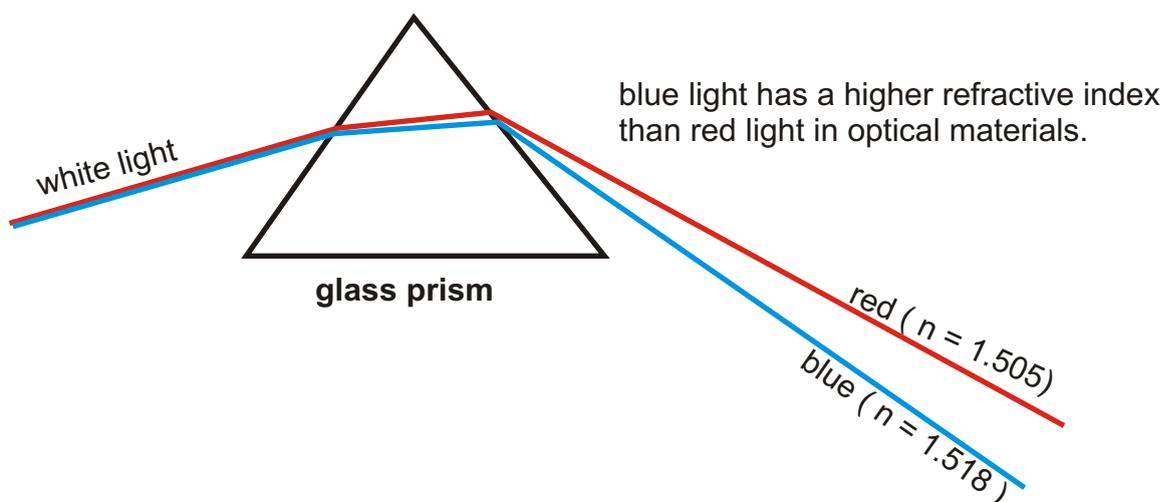
Absolute refractive indices for optical materials are quoted for different wavelengths of light. Refractive index is measured using the appropriate monochromatic light.

The different refractive indices also mean that the speed of light is also dependant on frequency. Optical fibre communication systems carry signals using monochromatic laser light. Carried by white light, the signal would be scrambled as the red signal raced ahead of the blue signal.

Example.
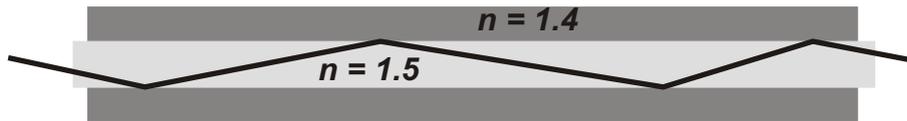
An optical fibre is made from two types of glass, one surrounded by the other. The inner glass has an absolute refractive index of 1.5, the outer glass has an absolute refractive index of 1.4. Calculate the critical angle for light passing through the inner glass.

$$n = \frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ material}$$

$$speed\ of\ light\ in\ material = \frac{speed\ of\ light\ in\ vacuum}{n} = \frac{c}{n}$$

for inner glass $\quad v = \dfrac{c}{1.5}$

for outer glass $\quad v = \dfrac{c}{1.4}$

refractive index for inner glass relative to outer glass $= \dfrac{speed\ in\ outer}{speed\ in\ inner}$

$$= \frac{c\,/\,1.4}{c\,/\,1.5}$$

$$= \frac{1.5}{1.4}$$

The critical angle $_c$ $\qquad \sin {}_c = \dfrac{1}{n}$

$$= \frac{1.4}{1.5}$$

$${}_c = 69^{\circ}$$

Intensity.

The intensity of radiation falling on a surface is the radiant energy per second on a square metre of the surface.

Intensity (I) is measured in watts per square metre ($W\,m^{-2}$)

Example: A laser has an output power of 2 mW. The cross-sectional area of the beam at a distance of 1 metre is 4 mm^2. **Calculate the intensity of the beam.**

$$I = \frac{P}{A}$$

$$= \frac{2 \times 10^{-3}}{4 \times 10^{-6}}$$
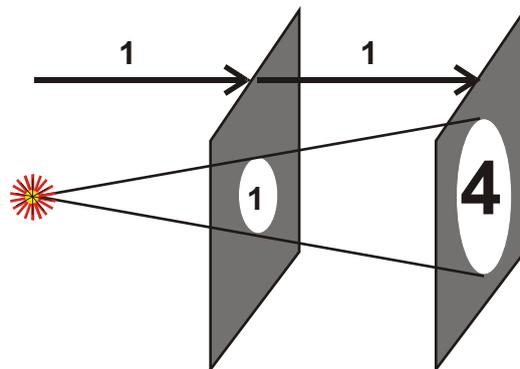
$$= 5 \times 10^{2}$$

$$= 500\ W\,m^{-2}$$

The Inverse Square Law

The intensity of radiation from a point source varies inversely as the square of the distance from the source.

$$I \propto \frac{1}{r^2}$$

$$I = \frac{constant}{r^2}$$

The constant is the same for all radiation from the same source.

double the distance and the same radiation is spread over four times the area.

Note: The inverse square law applies to point sources. The rule can be applied to large sources at large distance from the source where the source appears small to the observer.
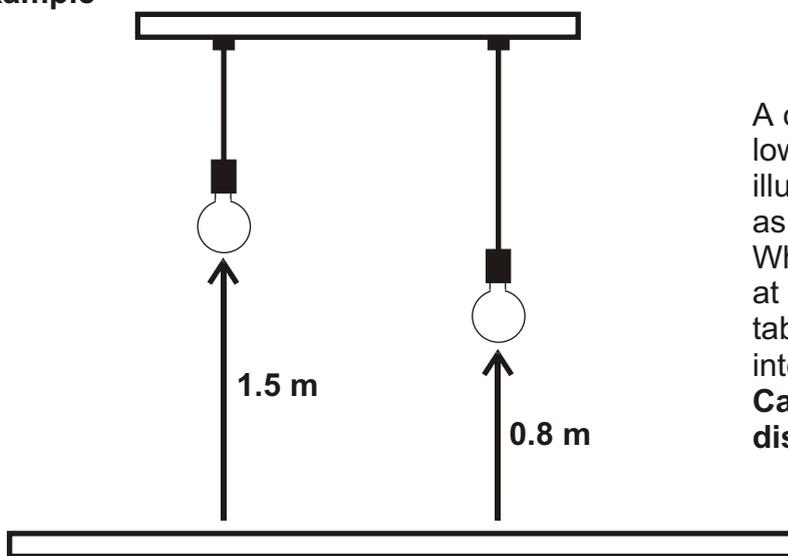
The inverse square law applies to all radiation; light, sound, radioactivity. It also applies to the force of gravity and the electric forces.

Experimental confirmation.

photodiode

r
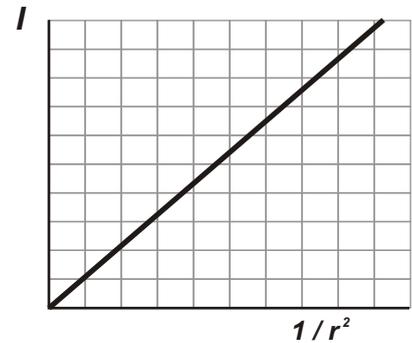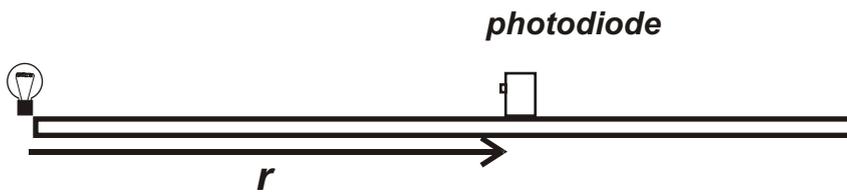
Readings are corrected for background light intensity.
Photodiode is set up in photoconductive mode where the
metered output varies directly as the intensity.

Example

1.5 m

0.8 m

A ceiling lamp can be raised or lowered to provide table top illumination at differing intensities as required.
When the lamp was positioned at a distance of 1.5 m above the table a light meter recorded an intensity of 0.35 Wm^{-2}.
Calculate the intensity at a distance of 0.8 m.

$$I = \frac{constant}{r^2}$$ *assuming lamp acts like a point source.*

$$0.35 = \frac{constant}{1.5^2}$$

$$constant = 0.35 \times 1.5 \times 1.5$$

$$= 0.79$$

for a distance of 0.8 m

$$I = \frac{constant}{0.8^2}$$ *same lamp, same constant*

$$= \frac{0.79}{0.64}$$

$$= 1.2 \ Wm^{-2}$$ *as expected, ½ the distance(approx) 4 times the intensity.*

Photoelectric Effect.

When certain metal surfaces are exposed to ultra-violet light, electrons are ejected from the surface. This effect is termed the photoelectric effect.

uv lamp

uv lamp

zinc plate

zinc plate

positively charged electroscope - no effect.

negatively charged electroscope - leaf collapses as electrons are lost.
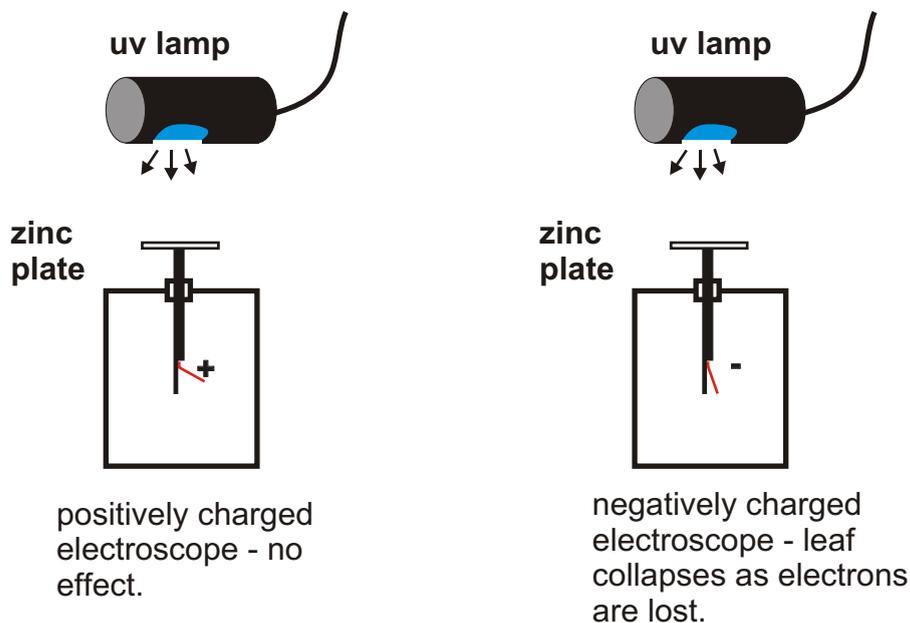
For a particular metal surface the effect can only take place for light with a frequency greater than a threshold frequency f_o. *Below this frequency the photoelectric effect will not take place even for light with very high intensity. Above this threshold frequency, the effect takes place, instantly and at all intensities.*

uv light

anode

(A)

- +

variable dc voltage supply.

photoelectric current

intensity
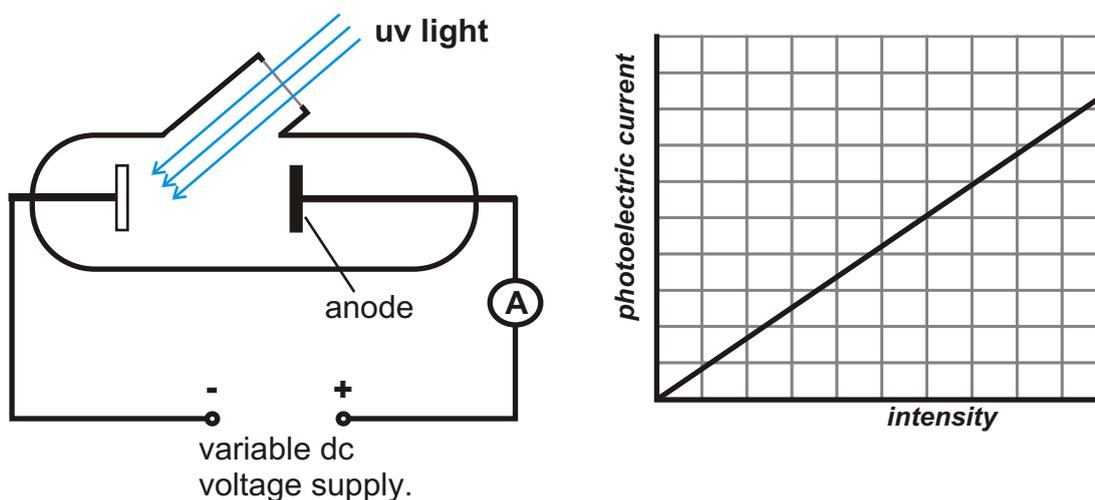
Using apparatus similar to that shown above, different metal surfaces are illuminated by different frequencies of UV light.. Any electrons produced are drawn across to the anode by the positive charge. The electrons then become part of a current which can be measured using the ammeter.

For frequencies above the threshold frequency, the photoelectric current produced by monochromatic light, varies directly as the intensity of light on the surface.

Light : Wave or Particle?

The photoelectric effect cannot be explained by wave theory. Light waves carry energy and it should be possible for that energy to be passed to an electron which can then escape from a surface. The only factor to be considered would be the intensity of the light! It should not depend on the frequency of the light.!

The photoelectric effect can be explained if light is regarded as a stream of particles: *Photons*. Each photon carries a package of energy which depends on the frequency of the light. The electron absorbs this package of energy. If the package carries enough energy, the electron can escape, if not, the electron cannot escape.

*Energy carried by a photon E = hf*

f = frequency of light in Hz

h = Planck's constant
= 6.63 x 10^{-34} Js

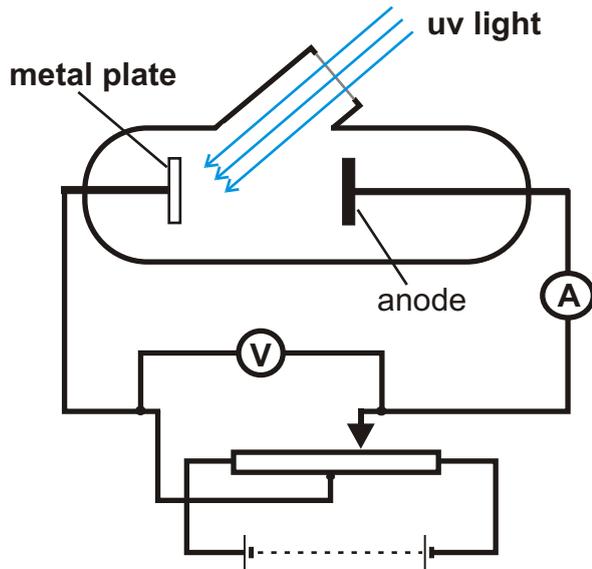**Example** Calculate the energy carried by a photon of light with a wavelength of 450 nm

$$E = hf$$

$$= \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{450 \times 10^{-9}}$$

$$= \underline{4.42 \times 10^{-19} \text{ J}}$$

Intensity and photons.

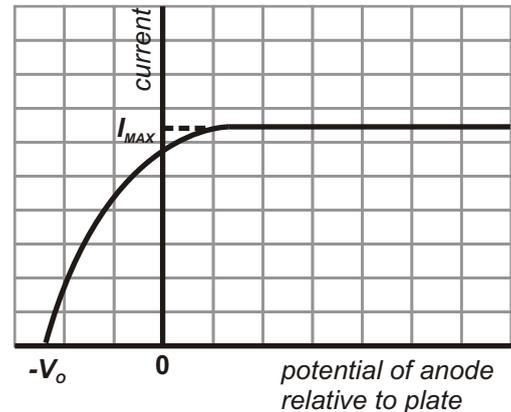If **N** photons with frequency **f** strike 1 square metre of a surface, the intensity of illumination **I** on that surface is given by

$$I = Nhf$$

N

E = hf

1 m^2

The Photoelectric Effect.

metal plate

uv light

anode
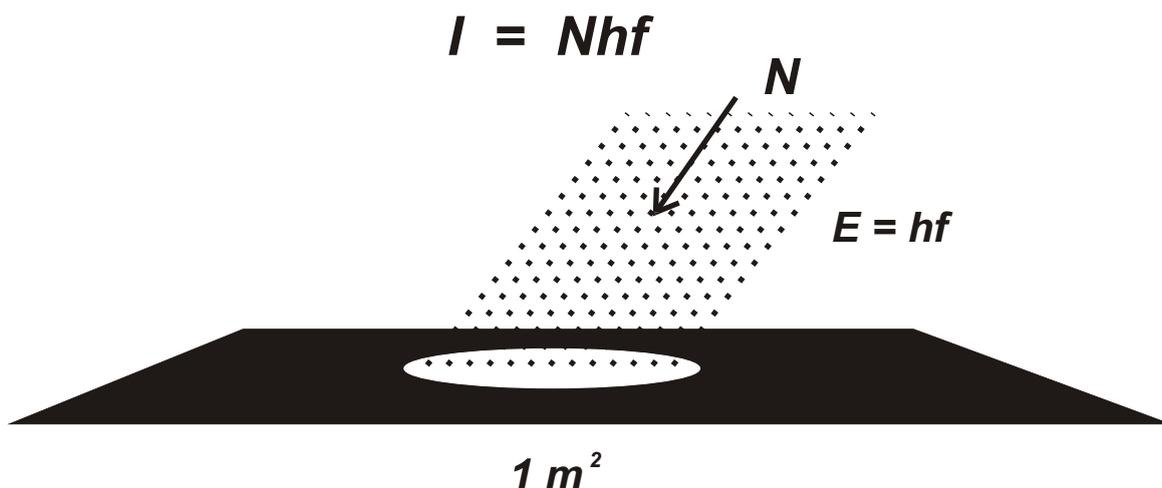
The voltage between the plates can
be varied or reversed as required.


The graph shows the photoelectric current
for light with fixed frequency and intensity.
*V_o is termed the stopping potential, the
reverse potential difference required to
stop the emission of electrons.*

The stopping potential indicates the
maximum kinetic energy of the ejected
electrons.

$$E_K = eV_o$$

When the stopping potential is plotted
against the frequency of the incident
light it is found to be directly proportional
to the difference between the frequency
of the light and the threshold frequency f_o

$$E_K = h (f - f_o)$$
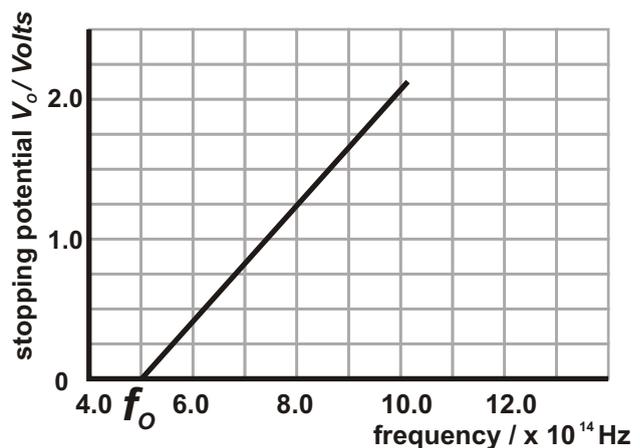
usually written $E_K = hf - hf_o$

E_K is the maximum kinetic energy of the electrons ejected from the surface

hf is the energy carried by the incident photon.

*hf_o is the work function of the surface (   ).This is the quantity of energy required
to free an electron from the surface.*

Example. A clean zinc plate is illuminated by monochromatic UV light with a wavelength of 250 nm. Calculate the maximum kinetic energy of the electrons emitted by the photoelectric effect (work function of zinc = 5.8×10^{-19} J)

$$E_K = hf - \text{work function}$$

$$= \frac{hc}{\lambda} - \text{work function}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{250 \times 10^{-9}} - 5.8 \times 10^{-19}$$

$$= \underline{2.16 \times 10^{-19} \text{ J}}$$

Example: Caesium has a work function of 3.12×10^{-19} J. Find the maximum wavelength of the light which will cause the photoelectric effect in caesium.

$$\text{work function} = hf_o$$

$$= \frac{hc}{\lambda_o}$$

$$\lambda_o = \frac{hc}{\text{work function}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3.21 \times 10^{-19}}$$

$$= \underline{619 \text{ nm}} \quad \text{(orange in the visible spectrum)}$$

The maximum wavelength is the wavelength of light with the threshold frequency f_o.

Emission Spectra.

mercury vapour

Line spectrum.

When electric current is passed through a gas, the gas glows, emitting light. When the light is viewed through a spectroscope, the resulting spectrum is composed of lines. The light is emitted as a series of single wavelengths represented by each line in the spectrum.
Each element has its own characteristic line spectrum which can be used to identify the presence of that element in a gas.
Each line represents an emitted photon of light. The spectrum represents the range of photons which can be emitted by a single free atom of the element; the atoms in a gas moving freely around.
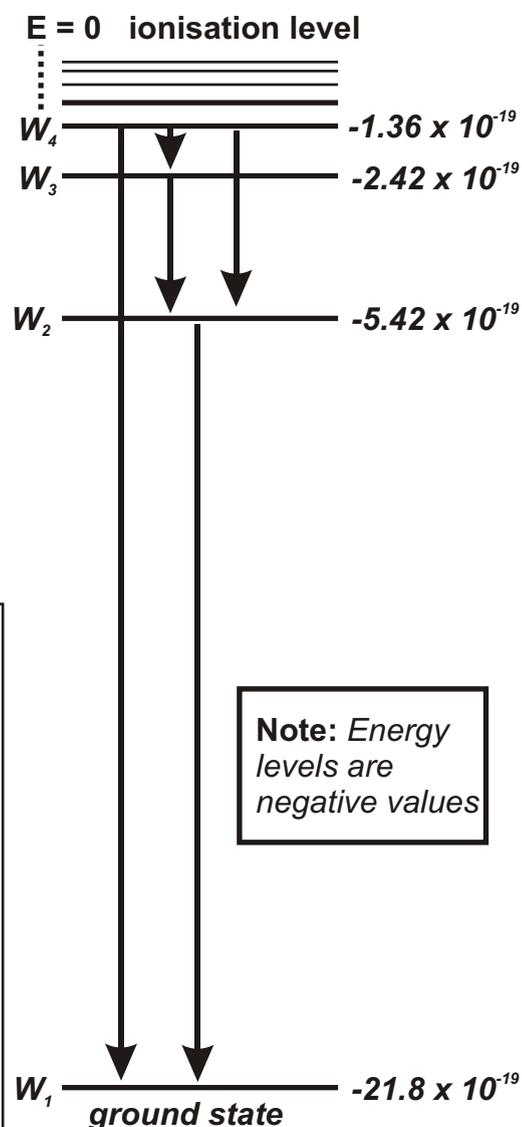
Energy levels.

Electrons in a free atom occupy discrete energy levels.

An electron falling from a higher level to a lower emits a photon of light with an energy equal to the difference between the two levels
Line spectra represent all the different possible ways an electron can fall back down the levels

The diagram opposite represents the energy levels in the hydrogen atom. The lowest energy level, W_1, is the *ground state.* This is where the electrons will be located in the unexcited atom. Only the lowest energy levels are shown. They get closer and closer as they get higher.

E = 0   ionisation level

W_4 ———————————— -1.36×10^{-19}
W_3 ———————————— -2.42×10^{-19}

W_2 ———————————— -5.42×10^{-19}

Note: *Energy levels are negative values*

W_1 ———————————— -21.8×10^{-19}
ground state

Example:
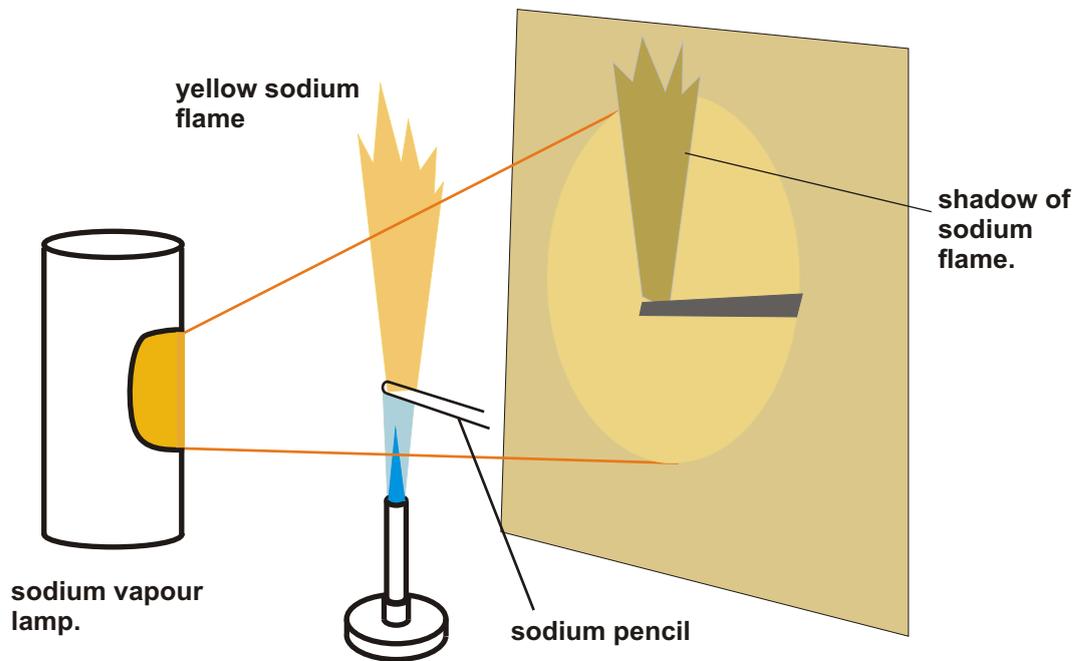When an electron falls from level W_4 to the ground state W_1, a photon is emitted with a frequency *f.*

$$hf = W_4 - W_1$$
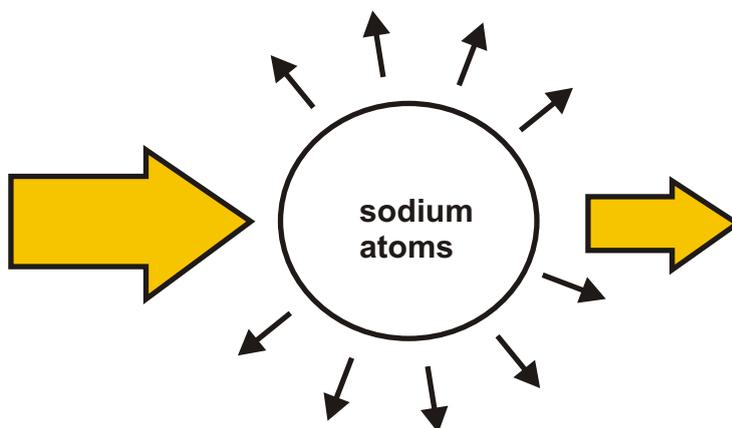$$6.63 \times 10^{-34} \times f = -1.36 \times 10^{-19} - (-21.8 \times 10^{-19})$$
$$f = \frac{20.44 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= \underline{3.1 \times 10^{15} \, Hz}$$

This is a photon with a wavelength of 97 nm lying in the ultra violet region of the electromagnetic spectrum.

---

yellow sodium flame

shadow of sodium flame.

sodium vapour lamp.

sodium pencil

When a sodium flame is illuminated by a sodium vapour lamp, a shadow of the flame is projected onto a screen as shown.

sodium atoms

The photons in the light from the sodium lamp are absorbed by the sodium atoms in the flame then spontaneously re-emitted. The re-emitted photons are emitted in random directions, mostly away from the original direction of the light. After passing through the sodium flame, the light has less photons and is dimmer than the light which by-passes the flame.

The electrons in an atom are restricted to a set of energy levels, so the electrons can only absorb an amount of energy which will allow the movement between levels. This means that an atom can only absorb photons of light which are identical to the set of photons which the atom emits as a line spectra.

Absorption Spectra

When white light is passed through a gas then the resulting continuous spectra has a series of dark lines corresponding exactly to the emission spectra of the gas.
This is due to the absorption of photons ant their re-emission in directions away from the beam.
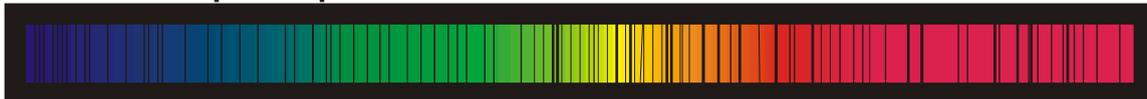
sodium vapour

**absorption spectra of sodium.**

stellar absorption spectra

The continuous spectra of stars, like the Sun, contain hundreds of absorption lines. These are a result of light from the surface of the star passing through a cooler gas atmosphere. The position and strength of the lines indicate the presence and concentration of various elements.

Stimulated Emission of Radiation.

Spontaneous emission of radiation is a random process where an atom captures a photon, holds on to it for a short period of time then re-emits the photon in a random direction. When the atom is in the excited state, it can be stimulated to re-emit the photon by an identical photon passing close to the atom. When emitted, the photon is in phase with the stimulating photon and is emitted in the same direction.

atom absorbs
photon

atom in excited
state

atom stimulated to
release photon
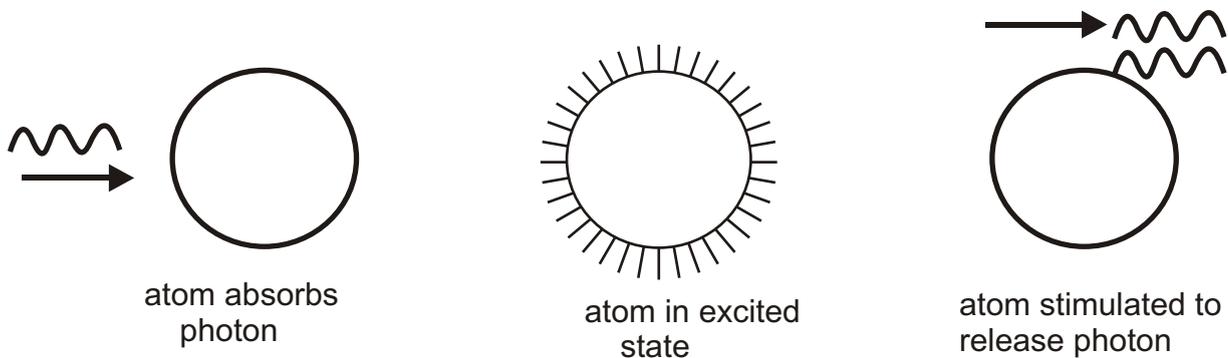
LASERS

If a large number of atoms are raised to the same excited state, a photon of light passing through them will rapidly collect a large number of identical photons as it stimulates the emission of photons from the atoms it passes.

ENERGY

full mirror

half-silvered mirror
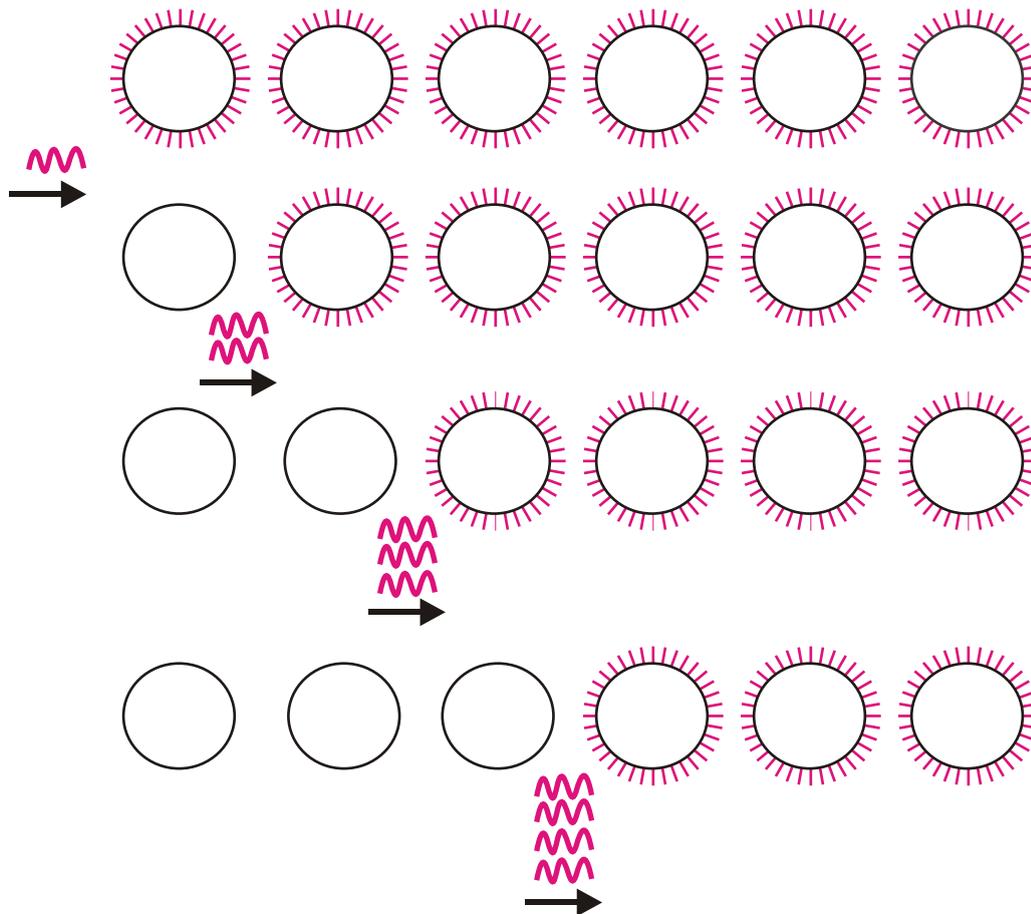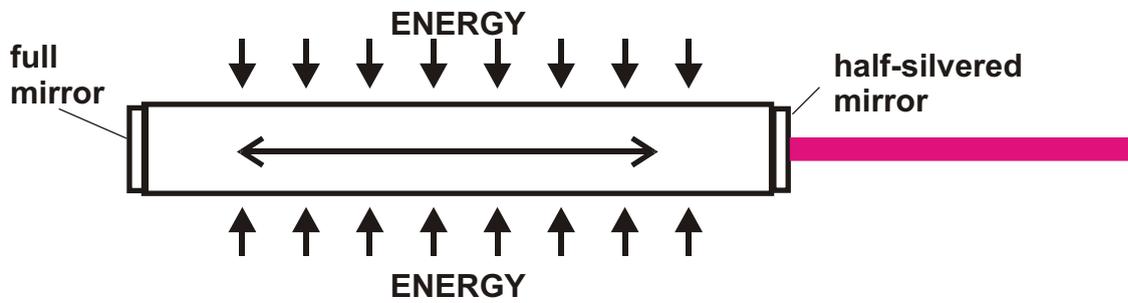
ENERGY

Lasers are constructed from material which contains atoms which can be excited to the same state. Energy, in some form, is 'pumped' into the material, raising most of the atoms to the excited state. Once in the excited state, any spontaneous emission of a photon will stimulate the emission of other photons.

Two parallel mirrors at either end of the laser ensure that photons travelling in the direction normal to the mirrors; along the length of the laser, are trapped in the laser. All other photons escape. As these photons travel along the laser they stimulate excited atoms to emit photons. Some of the photons are absorbed, but as long as more photons are emitted than absorbed, the intensity of the beam will increase.

One of the end mirrors is half silvered, allowing some of the light to escape as a narrow beam of intense coherent light.

Laser safety

Lasers produce a narrow beam of intense light which can permanently damage the retina of the eye. Even 'safe', low power lasers are potentially damaging.

Example   Calculate the intensity of a laser with an output power of 2 mW at a distance of 1 m when the diameter of the beam is 3 mm.

$$Intensity = \frac{Power}{Area}$$

$$= \frac{2 \times 10^{-3}}{x \,(1.5 \times 10^{-3})^2}$$

$$= 283 \; Wm^{-2}$$
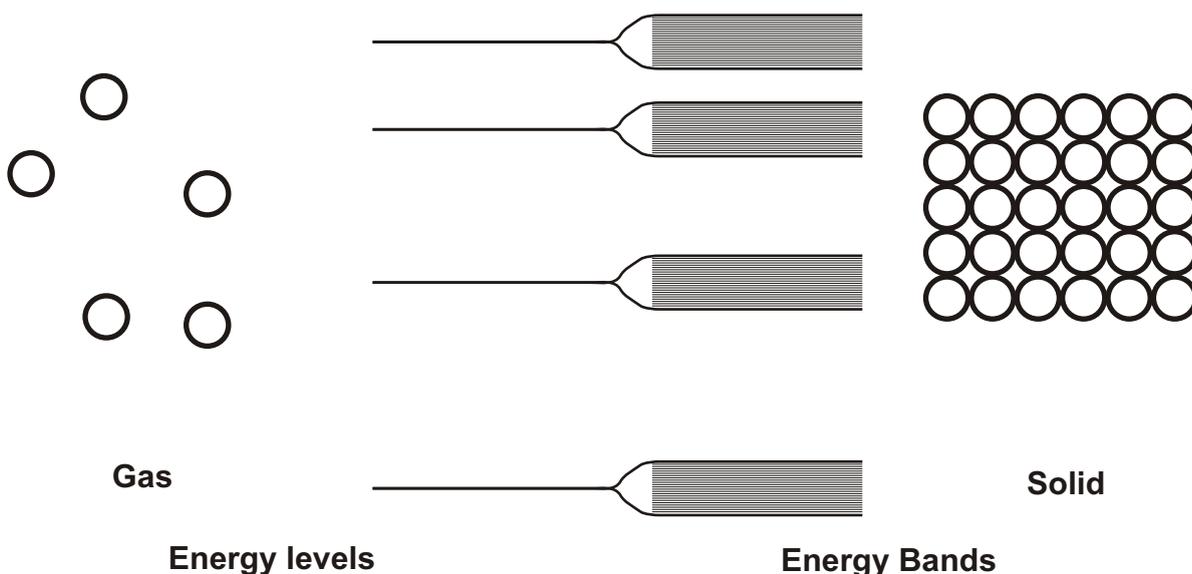
How does this compare with the intensity of a 100 W light bulb at 1m?

100 W

1 m
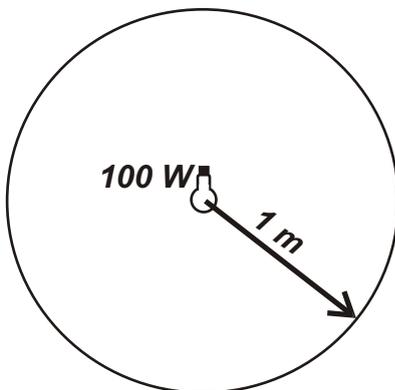
The power output of the bulb is distributed over the surface of a sphere with a radius of 1 m.

$$Intensity = \frac{Power\ output}{surface\ area\ of\ sphere}$$

$$= \frac{100}{4x \times 1^2}$$

$$= 8 \; W \, m^{-2}$$

This is a much lower intensity that the low power laser.

Energy Levels in Solids.
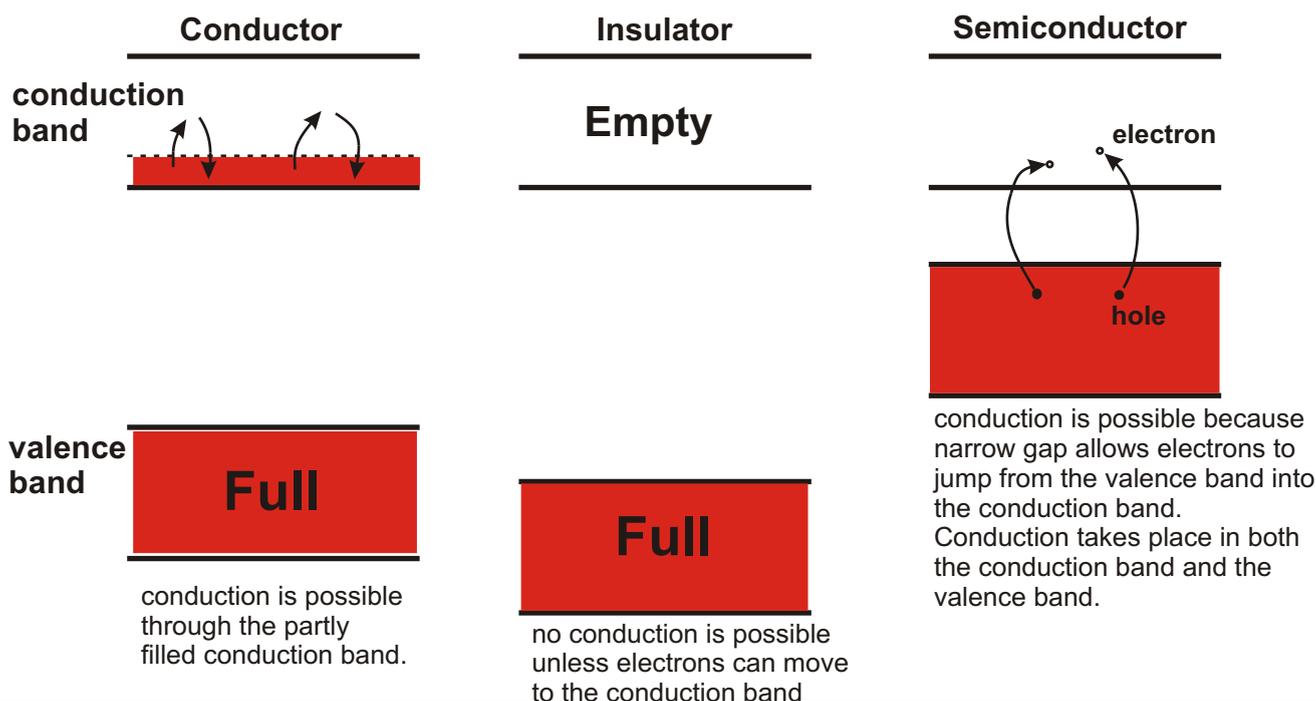

Gas   Solid

Energy levels   Energy Bands

The energy levels in free gas atoms are well defined. When the same atoms are formed into a solid, the atoms are linked together and the electrons can occupy a whole series of energy levels grouped into bands.
The electrons can move easily within the band but find it more difficult to move between bands.

Conductors, Insulators and Semiconductors.

Electrons in a solid can move between energy levels within a band. For electrical conduction to take place there must be electrons in a band and vacant energy levels to move to.
The ability to conduct electricity depends on the electron arrangement in the Conduction band and the lower Valence band. Electrons in the valence band are part of the interatomic bonding. The valence band is normally full.

conduction is possible because narrow gap allows electrons to jump from the valence band into the conduction band.
Conduction takes place in both the conduction band and the valence band.

conduction is possible through the partly filled conduction band.

no conduction is possible unless electrons can move to the conduction band

### Conductors, Insulators and Semiconductors.

Conductors. Metallic conductors contain many electrons which are free to move.

Insulators.   All the electrons in an insulator are tied into bonds and are not normally free to move. Given enough energy, the electrons will become free and the insulator will conduct. Insulators will start to conduct at high temperatures or at high voltages.

Semiconductors. Pure semiconductors (*Intrinsic semiconductors*) are insulators at very low temperatures. As they warm up, thermal energy frees electrons from bonds leaving holes in the bonding electrons. Bonding electrons can move through these holes to form a current. As the electrons move one way, the hole moves in the opposite direction like a positive charge.
The electric current is formed from the negative electron current and the positive hole current.
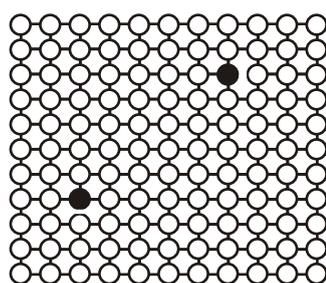
p and n type semiconductors.

Adding a small amount of other atoms to pure semiconductor material can increase the electrical conductivity of the semiconductor. This process is termed *doping.*
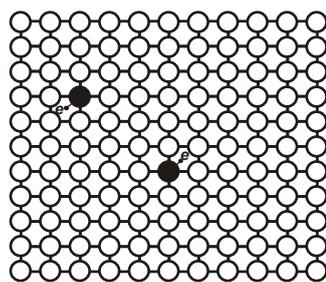
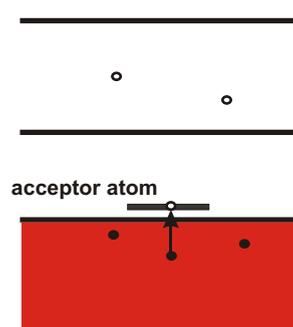

n - type
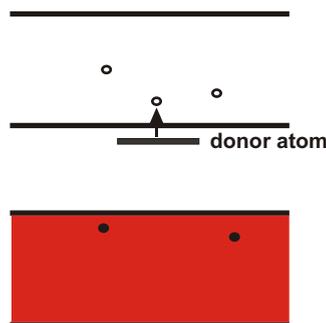
To produce n-type semiconductor, pentavalent atoms like phosphorus are added to the semiconductor. This provides extra free electrons for the conduction band.
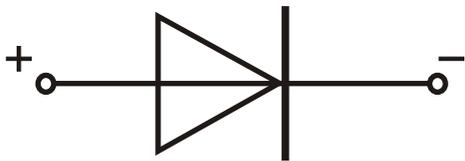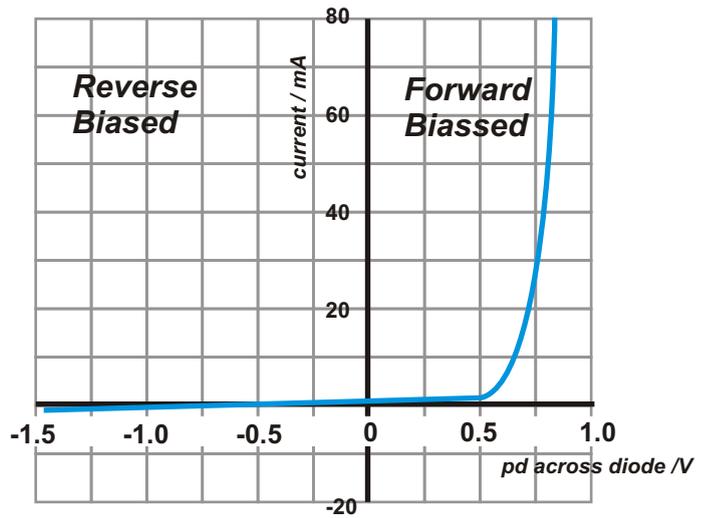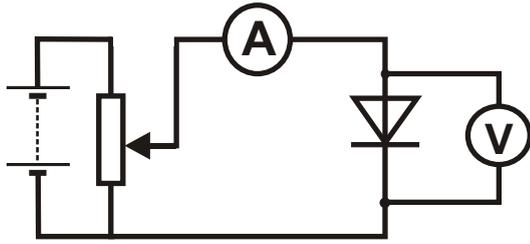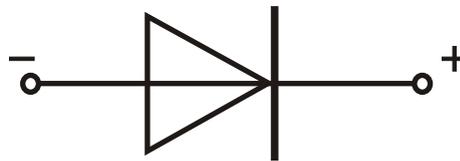

p - type

p-type is produced by adding trivalent atoms like aluminium. These create extra holes in the valence band.

Only a tiny amount of impurity is added. Doped semiconductor is termed *Extrinsic.*
In n-type semiconductor, the current is carried by negative electrons in the conduction band. In p-type semiconductor, the current is carried by positive holes in the valence band.
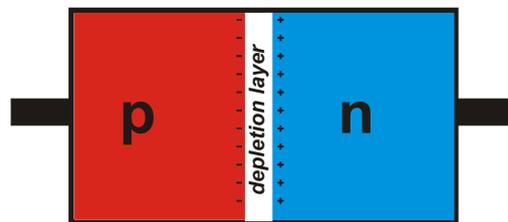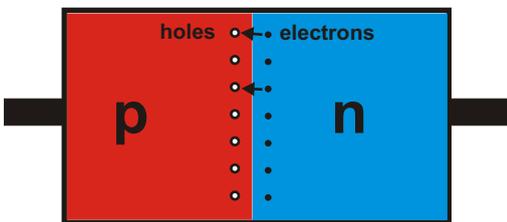
The p-n Junction Diode

Forward biassed : Current flows when pd is greater than 0.5 V. (in a silicon diode)

Reverse biassed : No current flows

electrons drift across the junction to fill holes. This has the effect of giving the p-type side of the junction a small negative charge while leaving the n-type side with an equal small positive charge. Once enough charge is built up, further electron drift will be prevented. This leaves a narrow region around the junction without any holes or electrons. This region is called the depletion layer.
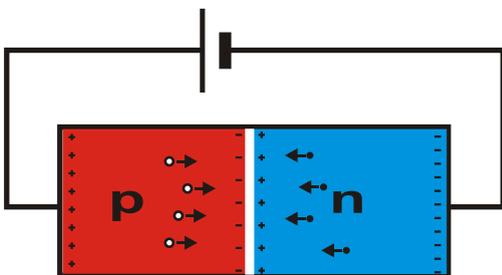
Forward biassed: positive holes and negative electrons flow towards junction. Depletion layer is reduced in width. A small pd is needed to overcome the effect of the charges on the junction.
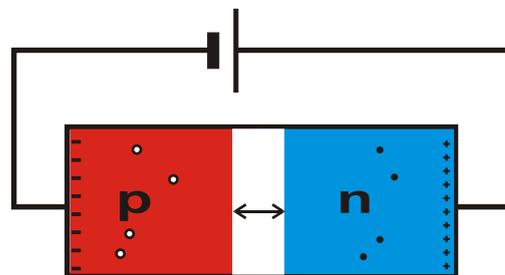
Reverse biassed: p side is made more negative and n side more positive. This prevents the movement of charge towards the junction. No current and depletion layer widens.

Light Emitting Diode (LED)

A light emitting diode emits light when it is forward biassed. A series resistor is always included in the circuit to limit the current flowing through the LED.
The emission of light is caused by electrons combining with holes at the p -n junction.
The electrons in the conduction band lose energy when they combine with holes in the lower valence band.
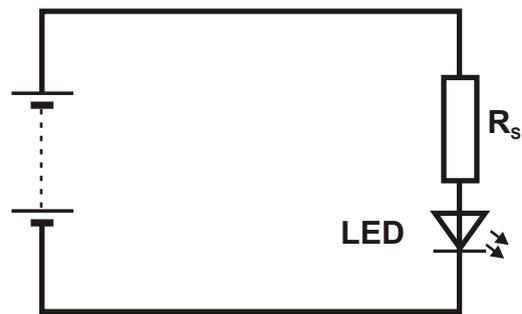

Light emitting diodes are made from materials like gallium arsenide or gallium phosphide.
LED 's are made which can emit red, orange, green or blue light. The p - n junction is formed close to the surface of the diode so that the light escapes.

The Photodiode.

A photodiode is a solid - state device used to detect light. Photons of light entering a p - n junction cause the separation of an electron from a hole.
A photodiode can be used in two ways.

Photovoltaic Mode:
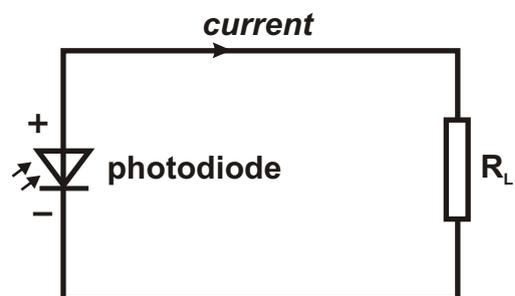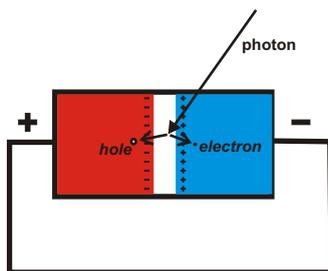
When a photon enters the depletion layer, an electron - hole pair is created. The space charge across the depletion layer draws the electron to the n - side and the hole to the p - side. The electron and the hole recombine when the electron flows to the p - side through an external circuit.
Photodiodes are used as a source of electrical energy in the form of the solar cell.
Solar cells are basic photodiodes with a large junction area.

Photoconductive Mode.
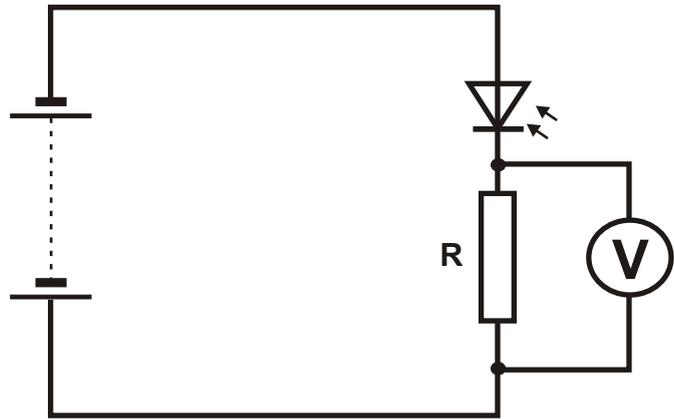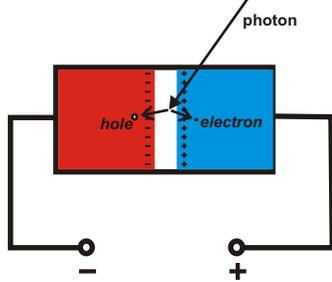
The photodiode has a reverse bias applied to it so that it does not conduct except when electron - hole pairs are produced by incident light photons. The current produced is directly proportional to the rate at which electron - hole pairs are produced which is directly proportional to the intensity of the incident light.
The currents produced are small, so the photodiode is operated with a large series resistor and the pd across the resistor measured ($V = IR$).

The current is a result of electron - hole pair creation and not the applied voltage, so is independent of the voltage as long as the voltage is below the breakdown voltage of the photodiode.

pin diodes.
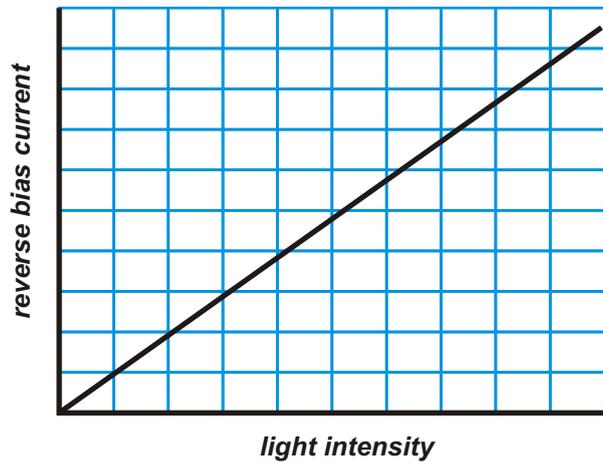
intrinsic semiconductor
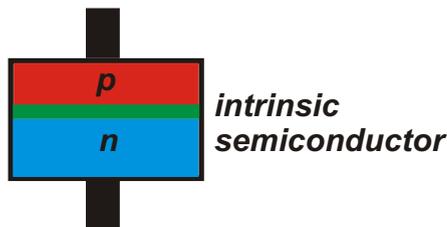
Pin diodes have an inbuilt thin depletion layer constructed from intrinsic (pure) semiconductor.
The depletion layer is very thin so that electron - hole pairs are swept up very quickly into the circuit.
This allows the diode to react extremely quickly to changes in light intensity.
These diodes are used to detect the digital signals passed through optical fibres.

# MOSFET (Metal Oxide Semiconductor Field Effect Transistor)
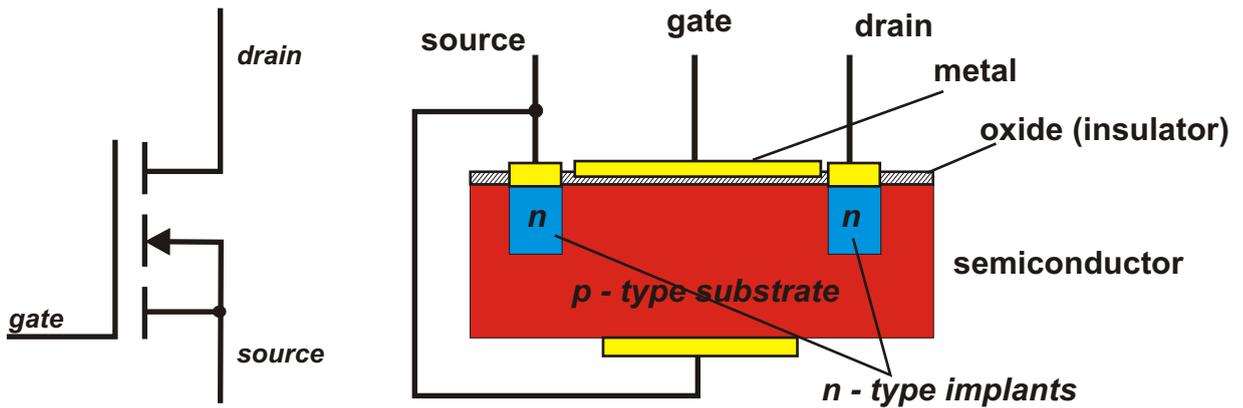
A MOSFET is constructed as shown in the diagram. A block of p - type semiconductor (the substrate) is the starting point.

Two n - type implants are diffused into the block. A coating of insulating silicon oxide is added. The oxide coating is etched away to expose the n – type implants.

Conducting metal contacts are added as shown. Note that the gate is insulated from the p - type block by the oxide layer. The metal contact on the bottom of the block is an internal connection. (this will be shown in the higher as an external connection).

How it Works

The drain is the positive terminal. With no voltage applied to the gate, current cannot flow between the source and drain. A reverse biassed p - n junction is formed at the drain implant.
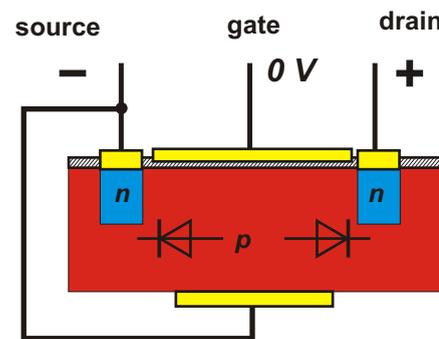
When a large enough (over 2 V) positive voltage is applied to the gate the electric field created between the gate and the bottom plate draws electrons to the gate. This forms a narrow channel of n - type semiconductor just below the gate. This n - channel connects the source to the drain allowing current to flow.
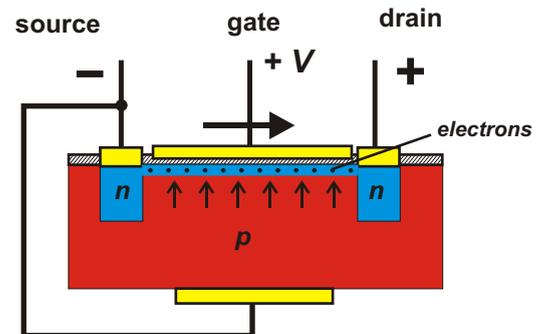
If the gate voltage is increased further, a wider channel is created with a lower resistance. A larger current can now flow.

The flow of current depends on the formation of an n - channel between the source and the drain. This device is termed an *n - channel enhancement MOSFET.*
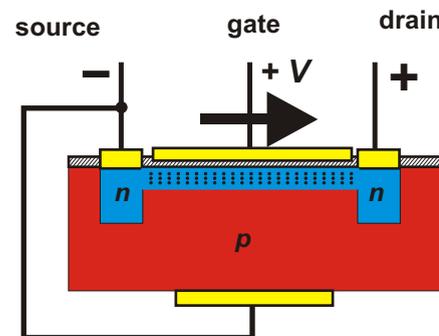
MOSFET will not conduct unless V_{GS} is above the threshold voltage: usually around 2 V.
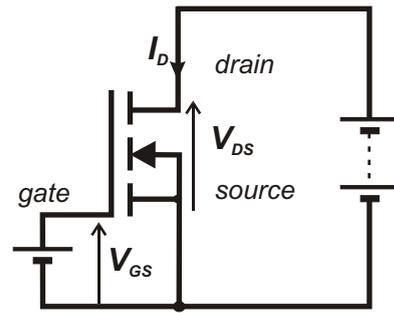Once V_{GS} is above the threshold, I_D, the drain current, can be increased or decreased by altering V_{GS} or V_{DS}.

A MOSFET can be used as a switch in the same way as a normal transistor. When V_{GS} is below the threshold value, the MOSFET is OFF. When V_{GS} is above the threshold value the MOSFET is ON.
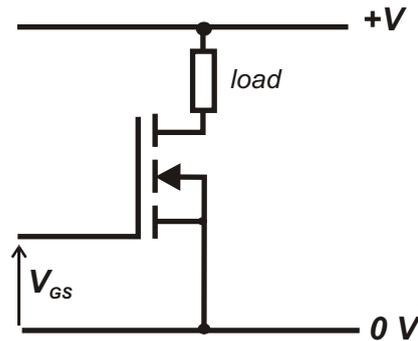
Once above the threshold voltage the drain current I_D varies directly as the change in the gate voltage V_{GS}. The MOSFET can also be used as an amplifier.

threshold
voltage
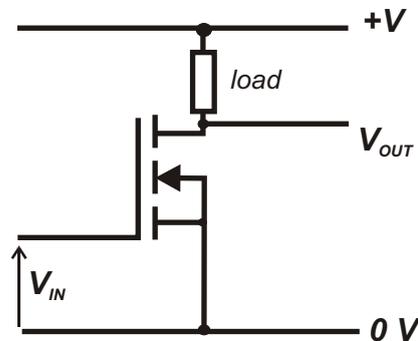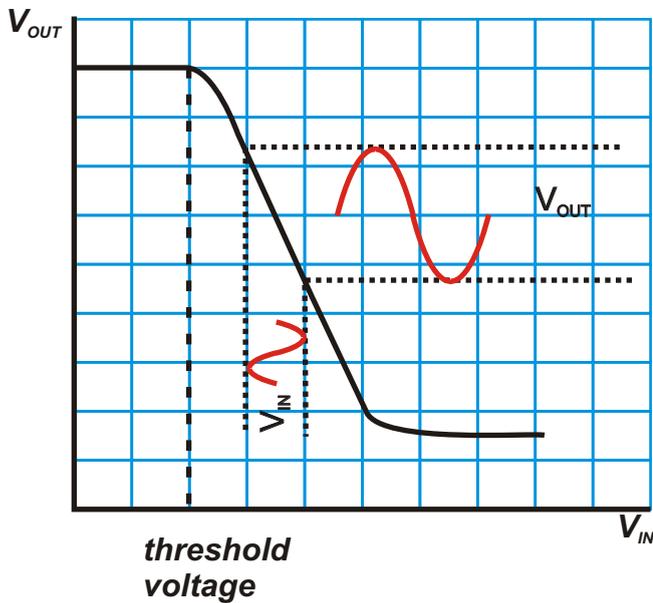
Geiger and Marsden, two research assistants of Ernest Rutherford, were directed by Rutherford to set up at experiment to observe how   particles were scattered by thin metal foils.

   particles from radioactive Radon gas were directed at the foils. The a particles, emerging from the other side were detected with a screen coated with zinc sulphide. This emits a tiny flash of light when struck by an   particle. The flashes of light were observed using a microscope. The number of flashes at different angles   , were noted.

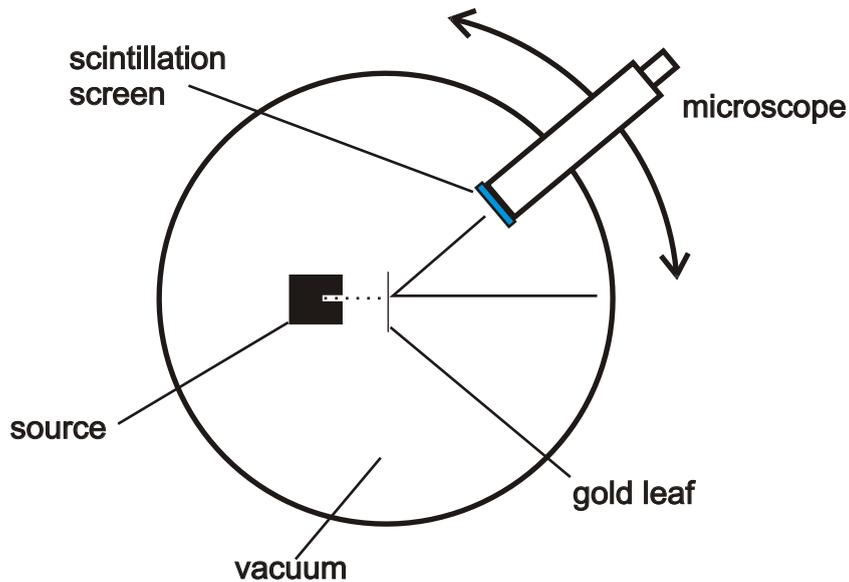*Results*   *(1) Most of the   particles passed straight through with very little deviation.*

(2) Some of the   particles were scattered through large angles. Of these approximately 1 / 8000 were scattered through angles greater than 90° .

Plum pudding model of atom

Rutherfords nuclear model
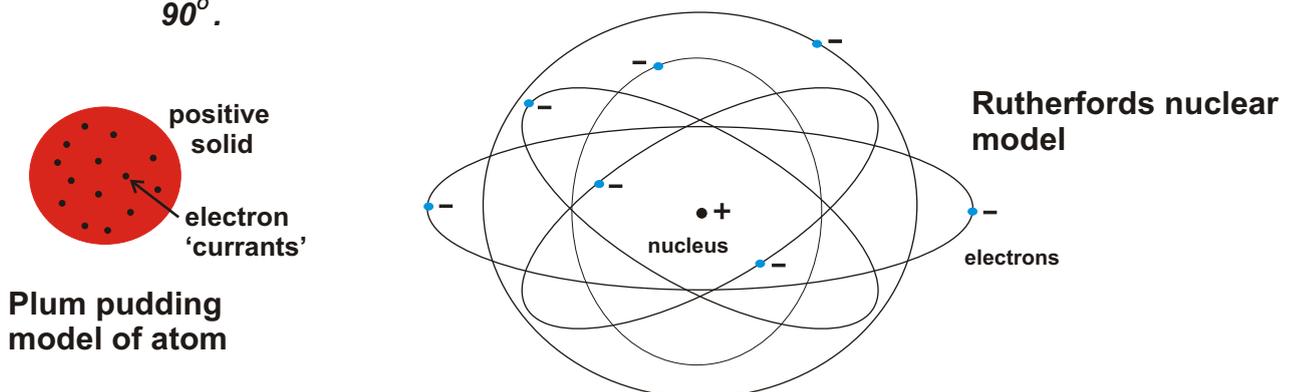
Rutherford explained the results by proposing a new model of the atom with almost all of the mass and all of the positive charge concentrated in a tiny nucleus. The rest of the atom consisted of electrons moving around the nucleus.

Most of the   particles would pass through the empty space round the nucleus. Only the few coming close to the tiny nucleus would be deflected by the large positive charge on the nucleus.

Radioactivity

A nucleus is made up of protons and neutrons. A proton holds a positive charge equal in magnitude to the negative charge on the electron, but is approximately 2000 times more massive. The neutron is similar to the proton but has no charge.

Atoms containing the same number of protons belong to the same chemical element. Atoms containing the same number of protons but different numbers of neutrons are isotopes of that element.

A nuclide is a particular nucleus and is designated by its element symbol, its atomic number and its atomic mass.
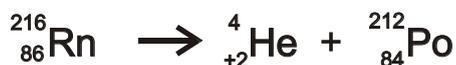
$$^A_Z X \qquad \text{examples} \qquad ^{14}_{6} C \qquad ^{235}_{92} U$$
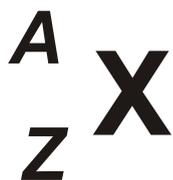
$$X = \text{chemical symbol}$$

atomic number Z = number of protons

atomic mass A = number of protons + neutrons

Radioactive atoms contain unstable nuclei. An unstable nucleus will decay; emit a radioactive particle to become more stable. The three types of radioactive particle we will consider are and particles.

particles are essentially helium nuclei. They consist of two protons and two neutrons

$$\rightarrow\ ^4_{+2} He$$
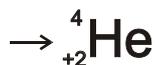
$$^{216}_{86} Rn\ \rightarrow\ ^4_{+2} He\ +\ ^{212}_{84} Po \qquad \text{particle decay is confined to heavier nuclei.}$$

$$^{176}_{77} Ir\ \rightarrow\ ^4_{+2} He\ +\ ^{172}_{75} Re$$
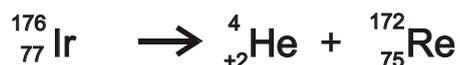
particles are high energy electrons from the nucleus. When a particle is emitted, one of the neutrons changes to a proton. The atomic number increases by one. The atomic mass is unaffected.

$$\rightarrow\ ^0_{-1} e$$

$$^{14}_{6} C\ \longrightarrow\ ^0_{-1} e\ +\ ^{14}_{7} N$$
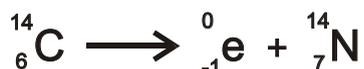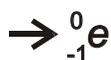
particles are bursts of high frequency electromagnetic radiation emitted by a nucleus to lose excess energy. particles are usually emitted along with and particles. particles do not change the atomic number or atomic mass of a nuclide.

Binding Energy

The protons and neutrons making up a nucleus require energy to keep them together. The energy required to do this is derived from some of the mass of the protons and neutrons. This shows up as mass defect; the difference in mass between the nucleus and the sum of the individual masses of the protons and neutrons which make up the nucleus.

Once the mass defect is known, we can work out the binding energy using Einsteins' famous relationship

$$E = m c^2$$

binding energy = mass defect x (speed of light)^2

Example. Find the binding energy of a helium nuclide $^4_2$He

mass $^4_2$He = 6.64249 x 10^{-27} kg
mass proton = 1.67295 x 10^{-27} kg
mass neutron = 1.67444 x 10^{-27} kg

mass 2 protons + 2 neutrons = 6.69478 x 10^{-27} kg

mass defect = mass of 2 protons + 2 neutrons - mass of helium nucleus

= 0.053 x 10^{-27} kg

binding energy = mc^2

= 0.053 x 10^{-27} (3 x10^8)^2

= <u>4.8 x 10^{-12} Joules</u>

As a rule the greater the binding energy per nucleon (protons, neutrons), the greater the stability of the nucleus.

When a radioactive particle is emitted from a nucleus, the binding energy per nucleon in the nucleus is increased. Some nuclei will emit a series of particles to reach the final stage of stability..

The energy carried by the emitted particle and the new nucleus is derived from the reduction in mass created by the radioactive decay.

Mass of radioactive nucleus = mass of new nucleus + mass of particle + energy / c^2
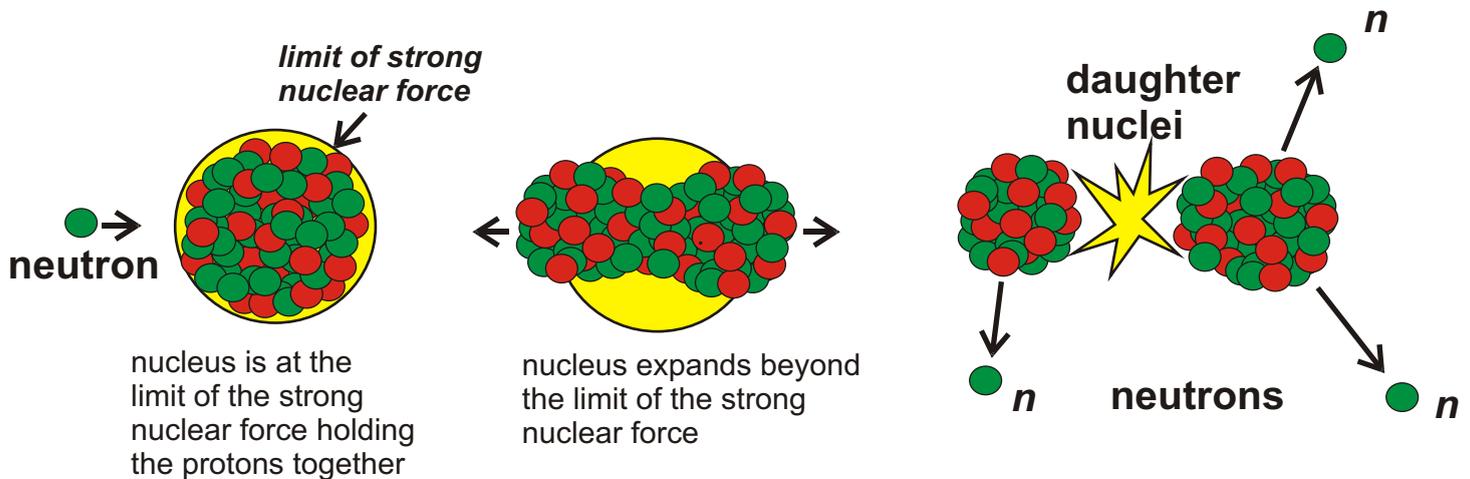

Nuclear Fission.
Some large nuclei are at the limit of the force holding the protons and neutrons together: the strong nuclear force. Any process which puts them over the limit will cause the nucleus to split in two, forming two new nuclei. The name for this process is nuclear fission. There are two different types of nuclear fission.

Spontaneous Nuclear Fission The nucleus simply breaks up in a process similar to radioactivity. This is a natural, uncontrolled, random event.

Induced Nuclear Fission The nucleus which undergoes spontaneous nuclear fission can be induced to undergo fission if it captures a neutron. When a neutron enters the nucleus, it becomes unstable and breaks in two.
When the nucleus splits, the new daughter nuclei emit further neutrons which can go on to cause more fissions. The process can run away into a chain reaction if the mass of material is large enough. In a nuclear reactor the chain reaction is slowed down by controlling the number of neutrons involved in the reaction.

One possible fission is shown below.

$$^{1}_{0}n \; + \; ^{235}_{92}U \longrightarrow \; ^{92}_{36}Kr \; + \; ^{141}_{56}Ba \; + \; 3^{1}_{0}n$$

$3.91847 \times 10^{-25} \text{ kg} \qquad 3.91539 \times 10^{-25} \text{ kg}$
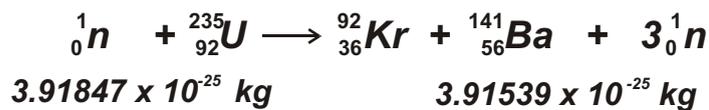
mass defect $= 3.08 \times 10^{-28}$ kg

energy released $=$ mass defect $\times c^2$

$= 2.77 \times 10^{-11}$ Joules
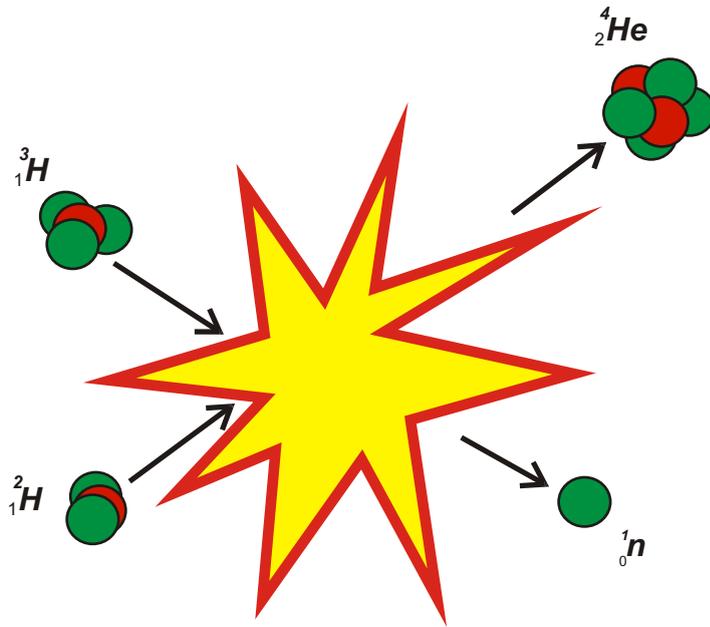
Nuclear Fusion.

Nuclear fusion takes place when two light nuclei are forced to combine and form a heavier nucleus. As with fission, the binding energy is increased and the resulting mass defect appears as energy.

$$_1^2H \ + \ _1^3H \ \longrightarrow \ _2^4He \ + \ _0^1n$$

$$8.347 \times 10^{-27} \ kg \qquad\qquad 8.316 \times 10^{-27} \ kg$$
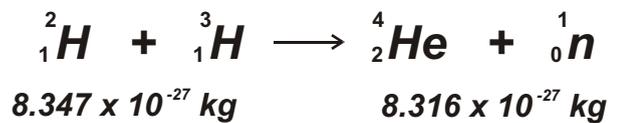
mass defect $= \ 3.1 \times 10^{-29} \ kg$

energy released $= \ 2.79 \times 10^{-12} \ Joules$

The energy released per fusion is lower than that for fission. However, the material involved is much lower in density than fissionable material, so the energy released per kilogram is much higher. Also, there is no critical mass involved so the energy released is unlimited. Fusion reactors offer the promise of unlimited energy supplies in the future if the problem of starting the fusion and containing the enormous temperatures can be solved.

Activity

The activity, A, of a radioactive source is the number of nuclei decaying each second.
Activity is measured in becquerels.

If **N** nuclei decay in **t** seconds, then the activity **A** of the source is given by

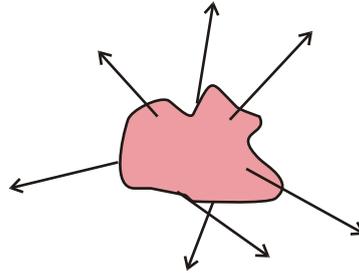$$A = \frac{N}{t}$$

Half Life

The activity of a source decreases with time, the time taken for the activity to fall to half its value is called the half life of the source.

Absorbed Dose.

Ionising radiation transfers energy to the material it is passing through.
The absorbed dose, **D**, is the energy transferred per kilogram of the absorbing material.
Absorbed dose is measured in grays (Gy) where one gray is an absorbed dose of one joule per kilogram.

Biological Effect.

The risk of biological harm from exposure to radiation depends on:

1. the absorbed dose,

2. the kind of radiation, eg   slow neutron.

3. the body organs or tissue exposed.

Quality Factor.

The quality factor, **Q**, is a number given to each type of radiation as a measure of the harm it can cause to living tissue.

The more ionising the radiation, the greater the quality factor.

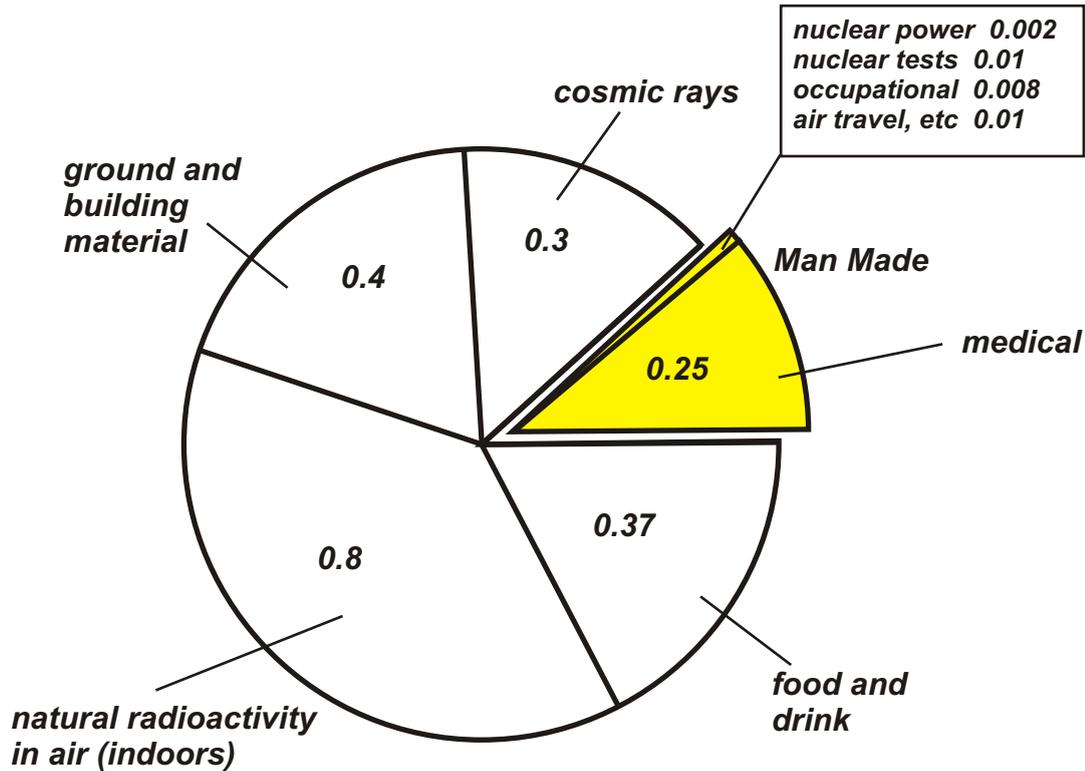| Radiation	Quality factor
	20
	1
	1
slow neutrons	2.3
fast neutrons	10

Dose Equivalent.

The dose equivalent, *H*, is a measure of the biological effect of ionising radiation on human tissue.

$$H = D \times Q$$

Dose equivalent is measured in sieverts (Sv)

The effective dose equivalent takes account of the particular organs and tissues exposed to the radiation.

nuclear power 0.002
nuclear tests 0.01
occupational 0.008
air travel, etc 0.01

cosmic rays

ground and
building
material

0.3

Man Made

0.4

0.25

medical

0.8

0.37

natural radioactivity
in air (indoors)

food and
drink

The chart shows the annual average dose equivalent of radiation received by someone living in the UK. Units are mSv / year
Most of the radioactivity we are exposed to comes from natural sources; cosmic rays, the ground around us, our food and the air we breathe. A small proportion come from man made sources: x-rays, residue from nuclear tests, air travel, nuclear power.

The annual dose rate for a citizen in the UK is around 2 mSv / year. The maximum recommended dose for the average citizen is 5 mSv / Year. Workers in nuclear industries are allowed a higher dose of 50 mSv / year.

Handling Radioactive Materials.

Shielding : Half Value Thickness

Counter

source **GM tube**

lead sheets

Thickness of lead / mm	Corrected count rate / counts/miute
0	380
1	294
2	232
3	191
4	150
5	122
6	98
7	78
8	60
9	47
10	39

rays

I $\frac{1}{2} I$ $\frac{1}{4} I$ $\frac{1}{8} I$

half-value thickness
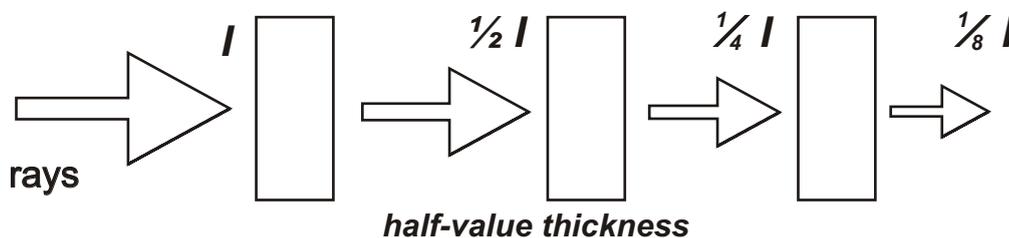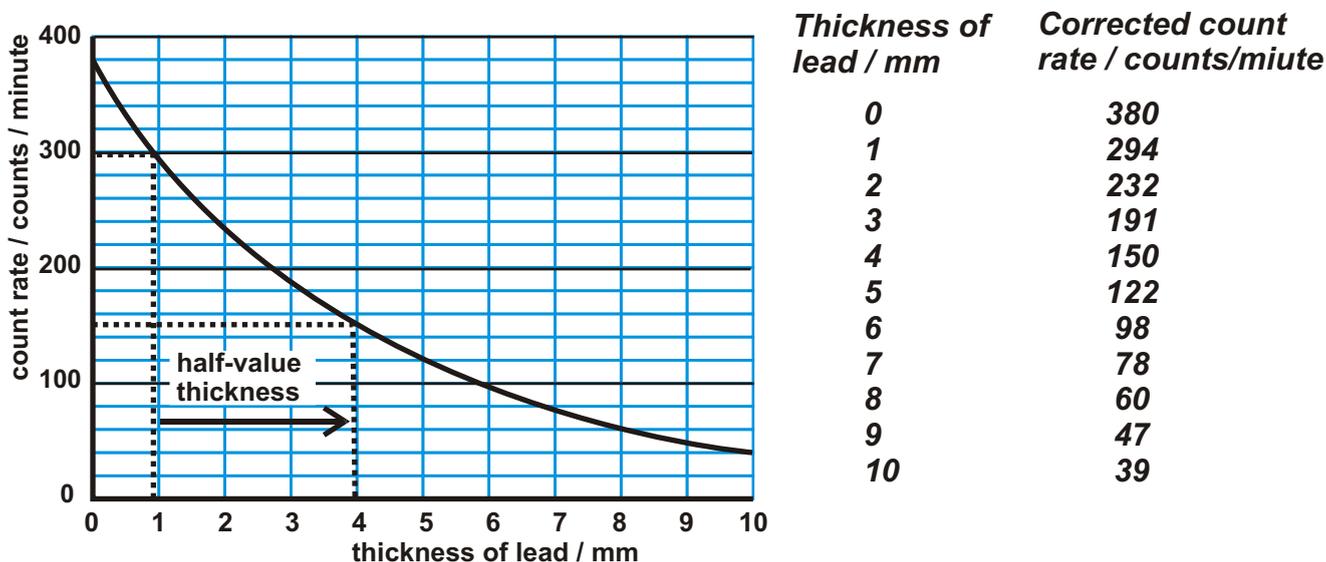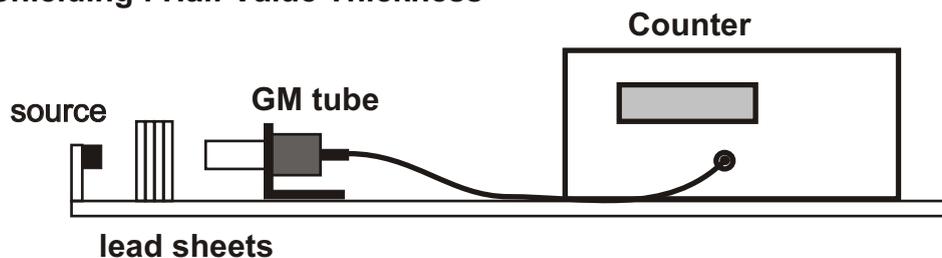
Gamma rays are absorbed by the material they pass through, so the intensity of radiation emerging from a material will be less than that entering the material. The thickness of material which will reduce the intensity of radiation to half of its value is termed the half-value thickness.

For shielding considerations, the half-value thickness of a material is the thickness of material which will reduce the dose equivalent of a source of radiation by half.

Example. The dose equivalent rate for a gamma ray source is 20 Sv per hour. The half-value thickness of lead for this source is 12 mm. What would be the dose equivalent rate with the source placed behind 48 mm of lead.

thickness / mm	Dose equivalent rate / Sv h^{-1}
0	20
12	10
24	5
36	2.5
48	1.25

The dose equivalent rate behind 48 mm of lead = 1.25 Sv h^{-1}

Inverse Square Law and Gamma Radiation.

Gamma radiation is a purely random process. The gamma rays are emitted in all directions; they spread out from the source in the same way as light. The intensity of gamma radiation obeys the inverse square law.
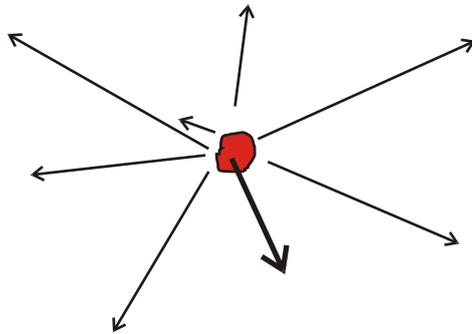
$$Dose\ equivalent\ =\ \frac{constant}{r^2}$$

Example. A gamma ray source has a dose equivalent rate of 50 Sv h^{-1} at a distance of 1 metre from the source. How far away from the source would a dose equivalent rate of 1 Sv h^{-1} be measured.

$$50\ =\ \frac{constant}{1}$$

$$constant = 50$$

$$1\ =\ \frac{constant}{r^2}$$

$$1\ =\ \frac{50}{r^2}$$

$$r\ =\ 7.1\ m$$

distance = 7.1 metres from the source

Radioactive sources are kept in shielded containers when not in use. When used, they are handled at a distance with tongs, in the case of   and   radiation, and very long tongs in the case of   radiation.

Everyone working with radioactive sources is fitted with a personal dosimeter to monitor their accumulated dose equivalent. These are checked regularly. If anyone has absorbed more than their allowed dose they are excluded from handling sources until their average accumulated dose falls back to a safe level.

Higher Physics

Errors and Uncertainties

Summary Notes

Errors and Uncertainties

All measurements are subject to error. We can never be sure that the measurement we have just taken is the true measured value.
Part of an experimenters' job is first, to estimate the true value, then estimate the uncertainty in that value.
Results are quoted

$$12.\ 83 \overset{+}{-} 0.02\ mV$$

estimated true value   *estimated uncertainty*

Types of Error.

Systematic Error

A systematic error is an error caused by poor technique or badly calibrated instruments.
This type of error causes readings to be consistently higher or lower than the true value.

Example: Force and acceleration for a constant mass where the runway used is not friction compensated.

systematic error
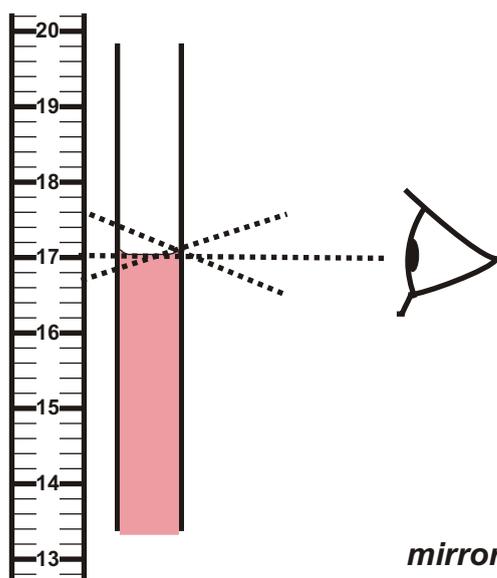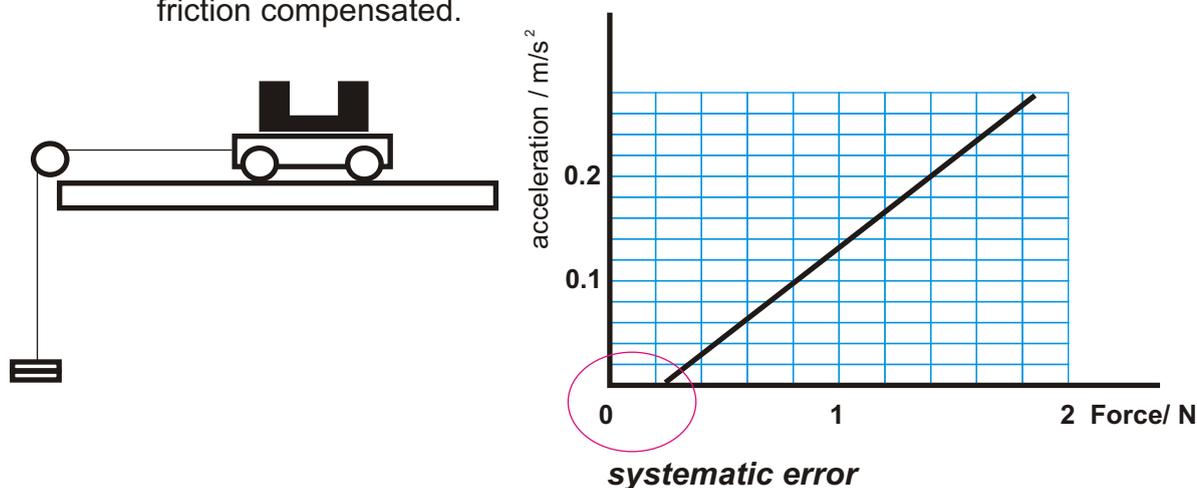
Parallax Errors
Scales should be read with the eyes level with whatever is being read. Analogue meters are sometimes provided with a mirror behind the pointer to prevent parallax errors.

mirror
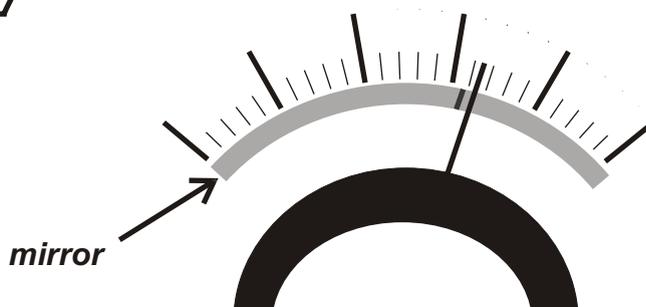
Random Errors

Random errors apply to repeated measurements. If we repeat the same measurement, we find that, despite our best efforts, differences will occur. The differences occur because it is impossible to recreate the exact same conditions for each attempt at measurement.

The measurements lie above and below the true value of the measurement. If we draw a distribution, we find that it is symmetrical about the true value

The best estimate of the true value of a set of repeated measurements is the mean or average value of the set

The uncertainty in the true value can be worked out statistically from the distribution. In Higher, however, we usually estimate this by working out the mean of the range in measurements.
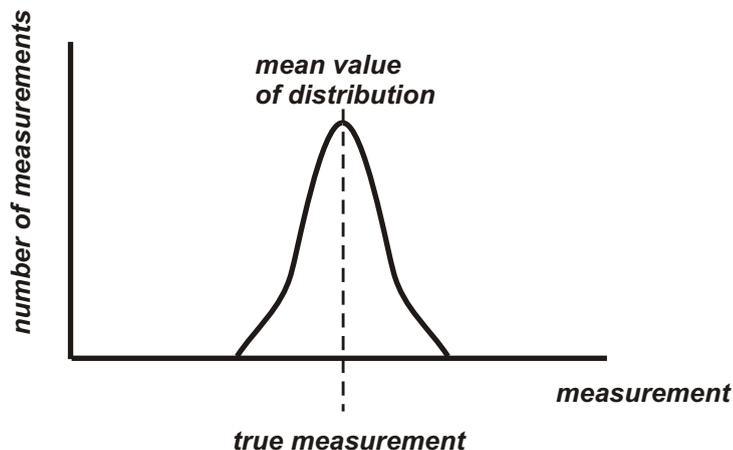
$$\text{uncertainty} = \frac{\text{maximum measurement - minimum measurement}}{\text{number of measurements}}$$

Example: Find the mean and random error in the following set of measurements:
5.63, 5.64, 5.70, 5.59, 5.63, 5.67, 5.62, 5.71, 5.65, 5.60.

$$\text{Mean value} = \frac{\text{sum}}{\text{no of measurements}}$$

$$= \frac{56.44}{10}$$

$$= \underline{5.64}$$

$$\text{random error} = \frac{\text{max - min}}{\text{no of measurements}}$$

$$= \frac{5.71 - 5.59}{10}$$

$$= \frac{0.12}{10}$$

$$= 0.012$$

$$\text{random error} = 0.02 \ (\text{rounded up})$$

$$\underline{\textbf{measurement} = \textbf{5.64} \pm \textbf{0.02}}$$

Reading Errors.

A reading error is an estimate of the accuracy of a single reading, where it is difficult to make repeated measurements.

Two types of error have to be taken into account. Firstly there is a calibration error. This is an estimate provided by the instrument manufacturer on the guaranteed accuracy of the instrument. This is expressed as a percentage of the full scale deflection. This error applies to all the readings.

The second error is an estimate of how accurate you can read the scale.

The scale shown opposite can be read to an accuracy of $\pm$ half a scale division. The instrument has a calibration error of $\pm$ 2% full scale deflection.

Suppose we have a reading of 1.12.

reading error = $\pm$ 0.05
calibration error = $\pm$ 0.02   *1.12 $\pm$ 0.05*

Suppose we have a reading of 4.23

reading error = $\pm$ 0.05
calibration error = $\pm$ 0.08   *4.23 $\pm$ 0.08*

Reading = 3.24
the last significant figure is an estimate

At bottom of scale, reading error is larger than calibration error. At top of scale, the calibration error is larger.
The larger of the two errors is used when quoting a measurement.

Analogue scales can be read to an accuracy of around half a scale division. Digital meters can be read to an accuracy of $\pm$ last digit.

7.985   **7.985 $\pm$ 0.001**

A digital meter is potentially more accurate than an analogue meter for the same calibration error as it can be read to more decimal places.

Percentage Error

A percentage error is the error in a measured quantity expressed as a percentage of the quantity.

Example : 6.3 $\pm$ 0.3   *percentage error* $= \dfrac{0.3 \times 100}{6.3}$

$= 5\%$

Error in Calculated Quantities.

Where a quantity is calculated from different measured factors, the error in the calculated quantity is found from the largest percentage error in the factors.

Example: In an experiment to measure the specific heat capacity of water, the following results were obtained.

$$mass\ of\ water\ = 505 \pm 2\ g$$

$$energy\ input\ =\ 6.55 \pm 0.05\ kJ$$

$$temperature\ rise\ =\ 3.1 \pm 0.5\ ^{\circ}C$$

$$c\ =\ \frac{E}{m\ x\ T}$$

$$=\ \frac{6550}{0.505\ x\ 3.1}$$

$$=\ 4184\ J\ kg^{-1}\ ^{\circ}C^{-1}$$

Quantity	% error
mass	0.4
energy	0.8
temperature	16

percentage error in calculated quantity = largest error in factors = 16%

This would give us an answer 4184 ± 670 $J\ kg^{-1}\ ^{\circ}C^{-1}$

Quoted answer 4.1 ± 0.7 kJ kg^{-1} $^{\circ}C^{-1}$